Scientific Research Base and Pilot Study for the Symphony Math Educational Software Program

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Introduction

As the U.S. makes the transition from an industrial economy to an information-technology economy, math skills are becoming increasingly important. Business leaders, politicians, academics and educators are in agreement that more sophisticated math skills are paramount to students entering the workforce in order to compete for well-paying jobs. There is less agreement, however, on the appropriate pathway for educators to help students achieve more sophisticated math skills. In this paper, we argue that there are four fundamental barriers that impede math learning. We present the rationale for the *Symphony Math* educational software program and propose it as a tool to help overcome these barriers.

Recently, mathematics education has received considerable attention in the national media and political landscape. President Bush emphasized the importance of mathematics education in the nation's global ability to compete in his January 31, 2006 State of the Union Address. The U.S. Congress commissioned the report *Rising Above the Gathering Storm* that illustrates how important strong mathematics skills will be for the success of the U.S. economy.

Unfortunately, there is no shortage of evidence that illustrates the discouraging mathematics' performance of U.S. students:

- U.S. students perform poorly in international tests of mathematics compared to other industrialized nations.
- The math performance of students lags behind their reading performance. In the 2003 TIMMS study, the U.S. placed 7^{th} in reading and 27^{th} in math.
- The National Assessment of Educational Progress reveals that nearly two-thirds of 8^{th} grade students are not proficient in math.

Barrier #1: Superficial Curricula

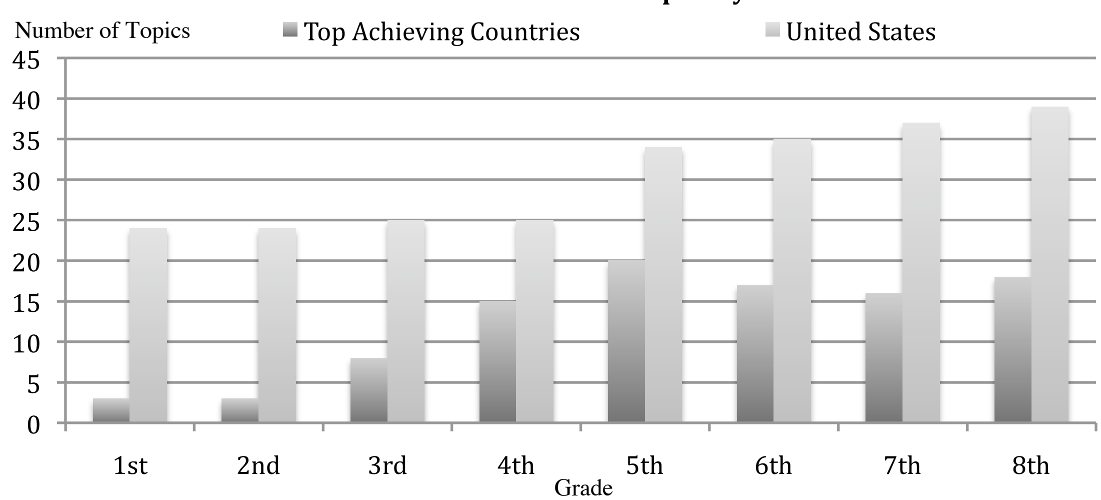
A major challenge to effective math instruction and successful math learning is the large number of topics that most states require students to learn. One researcher has referred to this phenomenon as the "mile wide and an inch thick" approach to math instruction. There are so many topics that most states require teachers to cover that there is not enough time for meaningful learning of any one topic. This leads to a teaching style that emphasizes the introduction of many concepts and skills without the time for in-depth mastery. The National Council of Teachers of Mathematics (NCTM) describes the situation:

> Teachers face long lists of learning expectations to address at each grade level, with many topics repeating from year to year. Lacking clear, consistent priorities and focus, teachers stretch to find the time to present important mathematical topics effectively and in depth (NCTM 2006, p. vii).

The 2006 Florida standards held teachers responsible for teaching 89 specific learning expectations for 4th grade math. An independent audit of the Georgia state curriculum found that it would take 23 years (not 12) to teach the topics specified for first grade through twelfth grade with anywhere near the level of depth for real learning to take place. Florida and Georgia have recently revised their standards to address this issue, but most other states have not.

We can see the U.S. emphasis of coverage versus depth when we compare the typical U.S. curriculum to the curricula of the top-performing countries in international tests of mathematics achievement. The chart below displays the number of topics taught for each grade for the typical U.S. curriculum (based upon the 1989 NCTM framework) and the typical curriculum of the top six performing countries. In the U.S., first-grade teachers are typically required to teach 24 topics. Teachers from the countries with the top math scores are teaching only three topics. The same is true for second grade. By only teaching three topics in first and second grade, these teachers are afforded more time to teach for mastery and ensure that their students have a profound understanding of the foundational concepts before moving on to other topics. U.S. teachers do not have this flexibility and often must move on to the next topic, regardless of whether their students have mastered previous topics.

Chart 1: Number of Math Topics by Grade



Let's look at the early fractions concepts as an illustration of how this plays out. In the U.S., most states begin teaching fractions in kindergarten or first grade and provide instruction on the topic yearly. The top math performing countries do not introduce fractions until third grade. The U.S. students are exposed to fractions from an earlier age and receive instruction on the topic yearly. The top-performing countries introduce fractions several years later but they spend more time on fractions and explore them in more depth. By third grade, they have also spent more time establishing a solid understanding of numeracy concepts that provide the foundation for understanding fractions.

We can look at a fourth-grade fraction question from the *Trends in International Mathematics and Science Study* test to see how these differing approaches play out over time: "There are 600 balls in a box, and 1/3 of the balls are red. How many red balls are in the box?"

Eighty percent of the students from the top math-performing countries answered this question correctly. Only thirty-eight percent of the U.S. students answered correctly, despite having been introduced to fractions three years earlier. The poor performance of U.S. students with fractions suggests that introducing too many concepts in the early grades and reviewing them each year is an inferior approach compared to introducing fewer concepts and devoting more time to mastering each one. This shows that the "mile wide and an inch thick" approach to math instruction is not serving U.S. students well.

The "mile wide and an inch thick approach" to teaching math creates several specific problems. First, the long list of learning objectives suggests that each objective is an end point in and of itself. The long list of objectives indicates that educators must teach each objective in an isolated fashion. It leads textbook publishers to focus on each topic as a separate unit in order to ensure that all topics are addressed. This checklist approach to math education diminishes the interconnected and hierarchical nature of mathematics. A student might study fractions on Monday, time on Tuesday, and addition problems on Wednesday; students learn about topics one after another without recognizing that there is an interrelationship between them.

A second flaw in the "mile wide and an inch thick approach" is that it often treats all of the objectives on the list as if they were of equal importance. Clearly, some mathematical concepts and skills are foundational to later learning while others are more peripheral. The long lists of objectives present all of the topics as if they were of equal value and thereby deserve equal emphasis during the school year. This, of course, is not true. For example, understanding place value is fundamental to the majority of math learning that follows and is a complex concept that takes considerable time to master. Learning the names of shapes is not nearly as complex; it does not take as long to learn and should not be given the same amount of classroom time or emphasis.

This checklist approach to math education has produced several negative outcomes. First, teachers are finding that they have to repeat instruction of key topics throughout the grades. Topics that were covered in first grade need to be repeated in second grade because the topic was not covered sufficiently for the learning to carry over to the next year. Second, students are performing poorly in algebra. The broad and superficial approach to math instruction is not providing students with the conceptual foundation that they need to succeed in algebra. Third, low-income students appear to be particularly negatively affected by the "mile wide and inch thick" approach to math education (NCTM 2006).

In response to the negative outcomes caused by superficial math curricula, NCTM now recommends that instruction should devote "the vast majority of attention" to the most significant mathematical concepts. There should be a focus on developing problem solving, reasoning, and critical-thinking skills. This will lead to the development of a deep understanding of important math concepts, mathematical fluency, and an ability to generalize.

Barrier #2: Math Wars

The "math wars" present another barrier to improving math learning. Math wars occur when a district, school or professional group become embroiled in a fierce debate about what is an appropriate form of math instruction. Reading wars are more commonly discussed, but math wars can be equally polarizing and destructive. Usually, the battle lines are drawn between a traditional curriculum that emphasizes procedures and memorization versus a progressive curriculum that focuses on conceptual understanding and interactive learning.

Math wars pose a barrier to improving math learning because they encourage the definition of extreme positions and the polarization of approaches that could otherwise be integrated. A school will embrace one side of the math war. After implementing that curriculum the school might conclude some years later that the curriculum is lacking in critical areas. Instead of moving towards a middle ground and implementing an integrated curriculum, the math wars' debate frames the choice as an either-or scenario. The school is likely to abandon the current curriculum entirely and implement one from the other camp of the math war debate.

Each approach in the math wars' debate has its strengths and weaknesses. The traditional approach offers a curriculum that is easier to implement and emphasizes systematic instruction. Its weaknesses are that it is not very engaging and some students have difficulty generalizing their workbook knowledge to every-day situations. The progressive approach provides curricula that include engaging activities that emphasize concepts, the ability to apply them in novel situations, and a comprehensive understanding of mathematics. The weaknesses are that these curricula are more complex to implement, require more teacher training and pre-class preparation, and some students have difficulty abstracting the mathematical principles from the interactive lessons.

The math wars have been taking place in one form or another for at least a hundred years. In the early 19th century, John Dewey advocated for hands-on learning, critical inquiry, the social education of the whole child, and was critical of rote learning. In the early 20th century, Edward Thorndike developed a method for systematically drilling and testing math facts. In the mid 20th century, Jean Piaget argued for the importance of the child constructing knowledge through hands-on problem solving with concrete representations (manipulatives).

While the debate has endured for some time, the reality is that these supposed ambassadors of the math wars did not stake out such extreme positions as the math war participants attribute to them. Thorndike, for example, also believed that conceptual learning was important; he was not narrowly focused on memorizing math facts. He believed that the pathway to conceptual learning was the solid knowledge of number associations. Piaget advised that in some instances, children should memorize certain educational content and he used math facts as an example. While he was an advocate of constructing knowledge and novel problem solving, he did not believe that children should always figure out what could easily be committed to memory.

Students and teachers do not need extreme, one-dimensional math curricula. They need sophisticated and differentiated approaches that meet their specific needs. Teachers and students are not asking to be placed in the middle of a culture war. Students want to succeed and feel good about what they have learned. Teachers want to be effective in helping students achieve these goals. The math wars are an obstacle to this goal because they encourage the development and implementation of math curricula that hyper-focus on some skills at the expense of others.

Both the traditional and progressive approaches have their strengths and weaknesses. However, math instruction needs to transcend the either/or dichotomy. Students need to have a differentiated approach where each student receives appropriate instruction. One student in a second-grade classroom may need help memorizing her addition facts while another student in the class is not ready to commit addition facts to memory because he first needs help in understanding the concept of addition. We need to assess each child's needs and learning style and teach them accordingly.

The National Research Council makes the case that both concepts ("conceptual understanding") and number facts ("procedural fluency") are important to creating a solid mathematical foundation. They also add that "adaptive reasoning," "strategic competence," and a "productive disposition" are important. Polarizing either/or debates pose a barrier to providing a more nuanced and comprehensive approach to math education.

Barrier #3: Implementation

The transition from educational theory, curriculum design and research to the implementation of these theories and curricula into something tangible within the classroom is another critical barrier that must be overcome to improve math learning. The challenge of implementation is great because the nature of mathematics instruction is complex and needs to be individualized for each student. How does the educational community provide high-quality mathematics education on a large-scale? How do we take the exciting methods and techniques from innovative math research studies and implement them faithfully across the country?

The challenge of implementation is particularly difficult to overcome at the elementary school level where classroom teachers are required to be experts in writing, reading, math and social development (among other things). Clements (2002), Ginsburg (2003) and Baroody (2002) outline the deep mathematical knowledge required for elementary teachers to perform effective instruction. This knowledge is not typically taught extensively in most teacher education programs, yet this knowledge is important to the effective teaching of elementary mathematics.

At the 2001 American Educational Research Association meeting, a team of researchers presented exciting results from an early elementary program called *Number Worlds*. The research results showed dramatic improvement in math learning by students at risk for math failure. The research team that conducted the study cautioned that they did not believe anyone with less than doctoral-level training would be able to effectively implement the program. The authors of the program insist that using the *Number Worlds* program is not a “piece of cake” for early elementary teachers. It requires considerable energy and effort “behind the scenes of classroom interaction.” (Case & Griffin; 1997)

Hiebert (1997) points to what he calls the teacher-researcher dichotomy. Researchers design the programs and conduct the efficacy studies and then expect teachers to faithfully implement these programs. There often is not enough support in the hand-off of the curriculum from the researcher to the teacher, who must fully implement the curriculum to ensure positive outcome.

A *Boston Globe* editorial described some of the challenge teachers had implementing a math curriculum in the Boston public schools. The article concluded that because teachers did not feel that the program was meeting all students needs, “. . . it might simply be the case that the city’s educators and students can no longer afford the luxury of a high-maintenance math curriculum.”

A New York Times article described a similar situation when New York City adopted a new math curriculum. The article cites a representative from the local teachers union who concludes, “Used by inexperienced teachers who are weak in math, they say the curriculum can be murky. And tutoring services say that they are seeing an epidemic of children coming to them for basic math instruction.” Writing about this situation in New York City, the *Christian Science Monitor* wrote, “New York’s teachers’ union praised the new curricula but worried that teachers would lack the skills to implement them.”

The challenges of implementing these sophisticated, research-based math curricula in Boston and New York illustrate the many challenges of implementation. Innovative, research-based and outcomes-based interventions exist that have tremendous potential to improve student math performance, but presently there is no reliable or systematic process to translate these exciting research results into concrete educational practice across the country.

Barrier #4: Memorization without Understanding

Another challenge in effective instruction and learning of mathematics is the ability for some students to memorize what they are “learning” in the early grades without fully understanding its meaning. Further complicating this issue is that much of the elementary math curriculum can be memorized with little understanding and still produce correct answers on commonly used worksheets and tests. With each passing year, these memorization strategies become less successful in delivering correct answers. The memorized strategies that were effective in answering questions relating to addition, subtraction, multiplication and division prove to be less successful in answering questions that demand a foundational understanding, such as fractions, long division, and decimals.

Memorization without understanding often begins with a student's introduction to formal math learning. Some students memorize the counting sequence of numbers from one to ten, or even to one hundred, without appreciating that each number represents an amount, or a quantity. Students can learn the counting sequence just as they do with the alphabet—memorizing a sequence of sounds and symbols. However, unlike the alphabet, the counting sequence represents a logical progression of proportionally increasing quantities.

In a kindergarten classroom, a student might be able to correctly answer worksheet tasks that ask questions about pictures of cookies or animals. When asked for the amount of a group of objects, the student counts those objects and records the number. But when asked another type of question, such as, "which number is larger, 7 or 9?" the student answers incorrectly because she does not recognize that numbers represent various quantities. This lack of understanding can go undetected by the kindergarten teacher because the curriculum that emphasizes coverage over depth does not provide the opportunity to explore more extensively into students' understanding and misunderstanding.

In first grade, a student can answer addition and subtraction questions by employing the strategy of counting up and counting down. Many students learn these operations by counting groups of objects or pictures of objects on worksheets. Some students do not extrapolate the underlying concepts of these operations. When they see addition, they count all of the objects, or maybe count up from the larger addend. When they are confronted with subtraction they count down. They can reliably produce correct answers to standard addition and subtraction problems without understanding the underlying parts-to-whole structure. If told they have seven marbles and are given two more, they count to seven and then count two more to reach nine. If they are then told they have two marbles and are given seven more, they count to two and then count seven more to reach nine. They have to solve the same problem by counting, because they have not internalized the concept of parts-to-whole relations that helps to understand that the whole will always be the same if the parts are the same, regardless of their order (commutative property).

In second grade, a student might commit the basic addition and subtraction number relationships to memory, but still might not understand the foundational concepts of these "number facts." She can immediately and accurately recall the answer when quizzed. She can even solve most word problems correctly. However, it is still possible that she has only a superficial understanding of the meaning of these operations. When confronted with a word problem, she has learned strategies that lead her to the correct answer; "John scored 7 goals. Sarah scored 3 goals. How many goals did they score altogether?" She sees the word "altogether", which she has learned indicates that she must count all the numbers together. She counts to seven and then counts three more to arrive at the correct answer of 10. Or maybe she has committed this number relationship to memory and summons the answer quickly.

This same student later encounters the following problem; "John scored 7 goals. John and Sarah scored 10 goals altogether. How many goals did Sarah score?" The student again solves this word problem based on keywords without understanding the concept. She sees the number seven, the number ten, and the word altogether. Using her adapted system for solving this type of problem, she combines the seven and ten and incorrectly concludes that Sarah scored seventeen goals. Since this is a more complex problem (missing addend), the student produces the incorrect answer and reveals her lack of understanding of the underlying parts-to-whole concept.

By third grade, this student is applying similar superficial strategies to multiplication and division. She may memorize the multiplication and division "number facts", but she does not appreciate the meaning of multiplication. She cannot explain what the term means and she does not understand its relationship to addition. She can produce the correct answer for multiplication problems but she cannot represent a multiplication problem using the repeated addition model, nor can she represent a division problem using the repeated subtraction model. She has some success with multi-digit addition and subtraction, but occasionally makes errors because she does not understand why she "borrows" and "carries" numbers. She does not understand the place value system, and that one hundred is composed of 10 tens or 100 ones.

By fourth grade, the strategies and counting models that have yielded mostly correct answers for her are no longer producing the same results. The procedures for long division are more complex and without her understanding of place value and the nature of number relationships, she has trouble remembering the proper sequence of steps to produce the correct answer. She is trying to add fractions, but the counting strategies she used for addition do not work for fractions. Although she was taught to solve addition problems by counting, she was not taught how to solve fraction problems by counting. For example, she was never taught how to count by one-thirds. Without a well-developed mental model of what addition means or a strong number sense, she struggles to reliably add and subtract fractions. Other topics, such as ratios, percents, decimals, and multiplication and division with fractions prove to be even more baffling.

Her case illustrates how some students can use superficial strategies and memorization (without understanding the supporting concepts) to appear to understand basic mathematical concepts in the elementary grades. The use of a broad and superficial curriculum that emphasizes coverage of many topics over in-depth understanding nurtures this guise of success. However, in middle school, these strategies are ineffective with more abstract concepts of fractions, long division and working with larger numbers. By this point, the pressure on the educators to cover a large number of topics specified for her grade level does not afford her the opportunity to go back and develop a stronger understanding of the foundational concepts she has not yet fully mastered.

Educational Software

Educational software has considerable promise in supporting classroom teachers' mathematics instruction. However, that promise has yet to be fully realized because the four barriers mentioned above limit the effectiveness of educational software programs just as they limit the effectiveness of classroom instruction. Superficial curricula, the polarizing effects of the math wars, the difficulties of implementation, and the tendency of some students to memorize without understanding, limit the potential power of educational software in becoming a more useful and effective tool.

State-mandated math standards that emphasize covering many topics as opposed to teaching fewer topics in greater depth encourage publishers of educational software programs to take the "mile wide and an inch thick" approach in their program design. There are a number of math software programs that provide instruction (or at least practice test questions) that align neatly with a state's curriculum guidelines. These programs cover many topics but do not attempt to develop comprehensive mastery of any of them.

Another common format of math educational software is "drill & practice," often in the format of flash-card style (memorization) activities. The basic focus is on the presentation and testing of addition, subtraction, multiplication and division facts. Games, cartoon characters and narratives are designed around the math-fact drills in order to help students remain engaged. These programs have the tendency to become entangled in the math wars debate because they focus on math-fact memorization and do not emphasize conceptual learning and applications, thereby promoting "success" through memorization but sometimes with little understanding.

There are math software programs that take the opposite approach from math-fact drill & practice. These are programs that provide more open-ended problems. In some cases, they only consist of on-screen manipulatives without an automated curriculum. These programs are designed to encourage the development of conceptual understanding and a broader range of skills than the drill-and-practice programs. Similar to the progressive classroom curricula, these programs require additional preparation from the teacher and require more background knowledge and training to properly implement them. Students need to be assigned to levels or units within the program and, in some cases, the teacher must monitor student progress and determine when the student should proceed to the next unit. Other programs require that the teacher teach the lesson with the program, thereby constraining the effectiveness of the technology to the knowledge and skill of the teacher.

Math software programs must be designed to thwart students' tendency to memorize math facts and procedures when they do not understand their meaning. Math software programs that offer practice on high-stakes test questions can show students how to solve many different kinds of math problems but often do not go deeply into the material to ensure that the student thoroughly understands the concepts. Math fact memorization software can help students' immediate recall of addition, subtraction, multiplication and division facts. But if students do not have a thorough understanding of the underlying concepts for these operations, this memorized material is superficial and does not provide the proper foundation for more abstract concepts that come in later years. Conversely, the more open-ended software programs that provide on-screen manipulatives and address conceptual knowledge are too dependent on the knowledge and training of the teacher to properly implement and support the success of the program on a broad scale.

Overcoming Barriers with *Symphony Math*

Symphony Math is an educational software program designed to help students develop a profound understanding of the most critical mathematical concepts, fluency with number relationships, and the ability to apply this knowledge to solving story problems. *Symphony Math* is a supplemental program that is used in conjunction with classroom instruction. The program provides students with the opportunity to practice and explore the most important math concepts in tremendous depth and in a variety of contexts. The program is designed to help teachers and students overcome the four barriers that prevent effective math instruction and learning.

Overcoming Barrier #1: Superficial Curricula
Symphony Math offers a focused curriculum that helps students to master the most fundamental elementary math concepts in tremendous depth. Research has shown that if instruction is focused on foundational concepts, in-depth learning can generally improve students' performance in related problems that were not specifically taught in the interventions. This focus on profound understanding of foundational math skills and concepts allows *Symphony Math* to promote the development of reasoning and generalization skills, which are difficult to develop with a superficial curriculum.

Overcoming Barrier #2: The Math Wars
Symphony Math avoids the typical «either/or» approach to math instruction by integrating a focus on conceptual understanding with activities that promote fluency. Students first establish an understanding of a fundamental concept, and then they work to master that concept through fluency activities where they must solve problems in five seconds or less. Students also apply their conceptual and fluency skills by solving story problems. The program addresses gaps in students' knowledge, whether they are concepts, math facts, or difficulties in solving story problems.

Overcoming Barrier #3: Implementation
Symphony Math is a fully automated software intervention program. A teacher can enroll her entire class into *Symphony Math* and let them work independently. *Symphony Math* shows the student how to use the program, tracks her progress and branches through the curriculum to ensure thtat she is at her appropriate level of challenge. The program interprets the student's actions to determine her level of understanding and directs her to the appropriate concept in its appropriate context in a way that would be very difficult for one teacher to do with each student in a large classroom.

Overcoming Barrier #4: Memorization without Understanding
Symphony Math helps students understand math concepts by continually challenging students to represent concepts with manipulatives, solve problems a variety of ways, and

to take their knowledge to deeper levels that go beneath the superificial surface levels of understanding. *Symphony Math* first introduces concepts with manipulatives so that students have a graphic representation of what the concepts «look like.» The program then bridges students to the abstract by coordinating symbols with manipulatives. The program also has students work with auditory math problems to learn the language of math and story problems to understand the concepts through narratives. Throughout the program, students are asked to solve a single problem with several different correct answers and make connections between concepts.

Symphony Math Overview

Symphony Math supports students in their development of a strong foundation of mathematical concepts and skills. The program provides an opportunity for students to develop and practice important mathematical ideas such as number conceptualization, part-to-whole relations, groupings of quantities, hierarchical groupings and composing and decomposing larger numbers. The program extends these fundamental ideas to operations such as addition, subtraction, multiplication, division, place value and multi-digit addition and subtraction. *Symphony Math* is intended for kindergarten through fifth grade students. The program can also be used with older students who have yet to develop a solid conceptual foundation in mathematical reasoning or are not fluent with basic number relationships.

Research in the field of cognitive development has mapped the landscape of important underlying concepts in mathematical learning and problem solving. The *Symphony Math* scope and sequence is based upon these insights. Major concepts presented in elementary math curricula are related to fundamental ideas or cognitive schemes that support them. Math understanding and learning is more effective and meaningful if instruction and practice are explicitly connected to these fundamental ideas. Crucial to understanding these fundamental ideas, is that they follow a developmental pathway throughout math learning. These concepts repeatedly emerge in successive levels of mathematical development and build upon one another in increasing complexity. Physical representations of the fundamental ideas provide a mechanism by which students can interact with these concepts, apply them in a variety of situations, and internalize a model that reinforces their meaning.

All students come to the classroom with some form of math understanding. The most effective way to improve math understanding is to identify where a student is within the levels of math development and to engage them at that level. It is important to connect instruction to a student's current level of knowledge and build upon it. For younger students, this means connecting current instruction to their «intuitive» or «informal» mathematics understanding, often by using manipulatives.

Students have the best chance of understanding fundamental ideas and internalizing number relationships if they have the opportunity to apply and represent these skills in a variety of contexts. Physical representations, number sentences, and word problems are some examples of different ways children can interact with and apply math ideas and procedures. The National Research Council and the National Council of Teachers of Mathematics recommend daily engagement with mathematics . *Symphony Math* is a supplemental intervention designed to complement instruction. The program provides the opportunity for students to work independently with fundamental concepts and number relationships on a daily basis both at home and at school.

Symphony Math Research Base

This section provides an overview of the key *Symphony Math* pedagogic techniques and explains their importance by connecting them to the scientific research. *Symphony Math* is a complex and sophisticated intervention program that is unique in its scope and methodology. The key program strategies and techniques for promoting a profound understanding of mathematics are rooted in a strong research tradition.

Multiple Representations of Concepts
Each *Symphony Math* module includes five activities representing fundamental concepts and number relationships in different contexts:

- Number Bars –Virtual manipulatives offer a visual representation of the concepts and number relationships.
- Bars & Numbers -- Tasks challenge students to coordinate manipulatives with numerals and symbols.
- Numbers -- Number sentences with symbols. If students struggle, the number bars appear in order to provide representational support.
- Auditory -- Number sentences are presented with spoken words. Students translate from words to numerals and symbols.
- Story Problems -- Tasks are presented in the form of story problems. Students translate stories into numerals and symbols and solve them.

These five different activities challenge students to represent concepts from different perspectives. Students concretely represent a concept with manipulatives that help them tangibly «see» what the concept means. The second activity uses manipulatives and symbols to help students connect symbols to representations of the meaning of the symbols. This helps students understand the meaning of the symbols and moves them away from the rote application of symbols. The third activity presents math problems with symbols. If a student makes a mistake or needs help, the manipulatives appear automatically to make the meaning of the symbol statement more concrete. The fourth activity presents math problems with spoken words, which students must translate into number sentences with symbols. This helps students learn the language of mathematics. The fifth activity presents story problems that put the concepts into action through narrative. These five activities provide different perspectives from which students work with all of the key concepts.

Multiple Solutions

Symphony Math emphasizes conceptual understanding by presenting problems that accept more than just one correct answer. Some tasks explicitly require students to provide up to three unique solutions. Students learn that math is more than right and wrong answers as they appreciate the connections between different concepts and number relationships. By requiring multiple solutions, the program promotes deductive reasoning and flexibility of thinking and helps students move beyond superficial memorization strategies. For example, the program will ask students to provide up to three distinct answers to problems such as:

- 3<?<8
- ?+?=10
- 10-?=?
- ?x?=12
- 18÷?=?
- ?+?+?=238

Solving problems like these (with multiple correct solutions), challenges students to think more deeply about the relationships between numbers and the relationship between operations.

Multi-Dimensional Branching

Educational software is often presented as a forward or backward march along a single path of learning material. The reality is that learning develops along multiple developmental pathways. *Symphony Math* is tailored to the complexity and uniqueness of each student. The program tracks student proficiency along three dimensions of learning and adapts to student fluctuations in their performance in real-time.

Symphony Math tracks each student's proficiency along the following three dimensions:

- The five different activities
- The progression of concepts (e.g., equals, greater than, less than and addition)
- The specific number relationships (e.g., 3<6, 3+4, 2*4)

The benefit of multi-dimensional branching is that it allows students to work within their developmental range with different representations and concepts. For example, a student might work on missing addend problems with manipulatives. This student may not be as strong with symbols and therefore will work on addition problems, which are easier than missing addend problems. The program might use an intermediate level of difficulty for number relationships, such as 4+3. The student may be less proficient with word problems than with symbols or manipulatives. Therefore, the addition word problems use the simplest number relationships, such as 2+1. The student practices a variety of these types of math problems. The multi-dimensional branching ensures that each problem will be at the student's appropriate level of developmental readiness and provides multiplechallenges that help the student make further connections between concepts, representations and number relationships. The complex branching algorithms and the detailed data tracking along the three dimensions provides a finely-tuned learning experience that is designed to the learning profile and developmental status of each student.

Conceptual Links

Symphony Math challenges students to make conceptual links between critical mathematical concepts. Making connections between concepts helps students see mathematics as an interconnected network of ideas instead of a checklist of procedures. The goal of this approach is to promote deeper understanding by providing students with problems that explicitly link corresponding ideas and concepts. When students connect a variety of concepts, they are more likely to remember and understand them because they are connecting new learning to their existing knowledge. There are several ways *Symphony Math* helps students make these connections.

1. The program introduces a concept with multiple representations. Working with the concept using multiple representations helps students make connections between the representations. For example, if a student makes a mistake with symbols, the manipulatives appear tangibly depicting why an incorrect symbol statement cannot render the correct solution.

2. *Symphony Math* uses the same manipulatives throughout the program. This helps students to make connections between the key concepts represented in each module and helps to consolidate their knowledge by building new learning upon existing knowledge. For example, the number bars are used in the Quantity module to help students understand that numbers represent different quantities. The larger numbers are associated with the taller number bars, and the smaller numbers are associated with shorter number bars. In the Addition & Subtraction module, this concept is reinforced by the continued use of the number bars. Every addition and subtraction problem with the number bars illustrates the new part-to-whole concepts that are introduced but also reinforces and connects to the previous concepts mastered in the Quantity module.

Time-Based Fluency

Symphony Math includes activities that challenges students to develop fluency in their recall of number relationships, or «math facts.» Once a number relationship or concept is understood with sufficient proficiency, the student will begin to solve number relationship problems based on the four operations in a timed problem-solving environment. A number relationship will appear on the screen, such as 10+7=? The student is challenged to answer as quickly as possible by selecting the appropriate number with the mouse. If the student answers correctly in less than five seconds, the number relationship is recorded as answered correctly in the database that tracks student progress. The student will then move on to a more challenging number relationship. If the student did not answer the task correctly, or answered it after five seconds, the number relationship will be recorded in the student database as needing more practice and will reappear in the near future. Students also work on number relationships auditorily. A student will hear, «three plus seven equals what number?» Students will see and hear number relationship problems in the format of missing addends, missing subtrahends, missing multipliers and missing divisors.

This approach to the development of mathematical fluency is unique in several respects. First, the program develops conceptual understanding before exposing students to time-based fluency activities. Second, the fluency activities consist of number relationships that go beyond the format of simple addition, subtraction, multiplication and division problems. The fluency activities in *Symphony Math* use problems in the form of missing addends, missing subtrahends, three part addition and subtraction, missing multipliers and missing divisors. This provides students with an opportunity to make connections between missing parts and missing wholes and develop fluency to a deeper level.

Mental Models

The development of mental models can be a powerful mechanism for understanding abstract principles . In *Symphony Math,* students work with virtual manipulatives (number bars) that provide a concrete representation of each concept. Over the course of using the program, students can internalize these concrete representations in the form of mental models. When new concepts are introduced, the student can integrate the new knowledge by connecting it to their mental model of the previous concept. The manipulatives are systematically connected to the abstract representation of concepts (symbols) and to the representation of concepts through narrative (story problems). This is designed to emphasize the meaning of each concept throughout their experience in each of the activity environments.

Detailed Data Tracking

The progress of each student is tracked in a database at a fine level of detail. *Symphony Math* records student progress through each activity level and records proficiency with specific number relationships (such as 7+5) or with specific concepts (such as addition) with in each of the five activities. The program uses these data to determine the proper sequence of tasks by branching the student to the tasks that are identified as specific areas of weakness within their developmental range of learning. These data are made available to teachers and administrators through a comprehensive reporting system. This detailed data tracking is a fundamental component to math instructional systems and is greatly facilitated by computer technology.

Developmental Approach

Symphony Math is a developmental intervention. The program seeks to find where a student's math-skill development is along conceptual pathways and joins the student at that point in their learning. This enables the program to provide problem-solving activities that meet the student at the appropriate level of skill or slightly beyond. As the student progresses, the program provides increasingly complex challenges along each developmental pathway. If the student struggles, help is automatically activated in the form of hints that lead the student towards a solution or it triggers the program to incrementally decrease the level of challenge in real-time. If the student succeeds on several tasks along the same conceptual pathway, she is rapidly promoted to the next level. By working with students developmentally, *Symphony Math* connects new knowledge to existing knowledge, which has been shown to be a more effective instructional technique than introducing content that is beyond the current knowledge base of the student.

Scaffolded Approach
Concepts in *Symphony Math* are introduced in a scaffolded structure. Each concept is mapped into a sequence of developmental levels, or «microlevels.» Students work through microlevels towards mastering a concept. When a student struggles, the multi-dimensional branching directs the student to the lower microlevels of the concept. As the student succeeds, she is directed to more advanced levels of the concept. Students who are moving through the program with a higher rate of success will branch from the highest microlevel of a concept to the highest microlevel of the next concept. When difficulty is encountered, the student is branched to lower microlevels of that concept. Scaffolding is also available in the form of a hint button that students can press when support is needed with a particular problem.

For example, early in the *Quantity* module students are introduced to the concept of «greater than.» In Activity Three, students solve problems such as 2<? with symbols. The student selects the answer from a row of eleven numerals (zero to ten) presented in random order. If the student makes a mistake or presses the help button, the problem is automatically reconfigured to the next lower microlevel. In this case, that means that the numerals that were randomly sequenced from left to right are now presented in order from zero to ten. Some young students do not know which numbers are bigger or smaller unless the numbers are presented in sequence from smallest to largest. If the student makes another mistake or asks for more assistance, the number bars appear automatically above the numbers. This allows the student to see that the number one is smaller than the number two and the number three is larger than the number two. If further help is required, the number bars will appear above the problem itself. This renders the problem in the same form as other problems in which the student has already achieved success.

Individualized
The individualized learning experience that *Symphony Math* offers simulates an individual tutoring experience where the program responds to the specific needs and actions of each student. *Symphony Math* tracks student progress at a fine level of detail in order to adapt to the specific needs of each student. The program adjusts to each student's level of conceptual understanding, learning style and content mastery. One student may be a visual learner who is strong with concepts but weak with number relationships. Another student may be more of a verbal learner who learns well through narrative but has trouble representing concepts visually. *Symphony Math* identifies these needs and provides the appropriate intervention. Each student progresses through the program according to their own needs and abilities. While this individualized style of learning is strongly supported by research, it is very difficult for one teacher to implement it with a large group of students who may represent a diverse range of learning styles and abilities.

Engaging

Symphony Math is designed to be intrinsically motivating. The program seeks to engage students by emphasizing the interesting patterns and ideas of mathematics. Students are challenged to make links and identify patterns in order to discover the inherent order and systematicity of mathematics. If students' attention is drawn towards discovering the fundamental nature of math and its applications, this will sustain student interest more profoundly than cartoon characters or interactive narratives. An overuse of cartoon characters and narratives in educational software suggests that the educational content is inherently disinteresting and therefore the characters and narratives are necessary to «sugar coat» the content. If we do not engage the interest of the students in the mathematical content itself, it is unlikely that we will promote the profound understanding of mathematics that we seek.

Keeping the challenge of the problems at the developmental level of the student is a key strategy to achieving high levels of student engagement. If the program is too challenging, students become anxious or frustrated and the engagement is lost. If the problems are too easy, the students may become bored and lose interest. The multi-dimensional branching of *Symphony Math* is designed to keep the level of challenge within the developmental range of each student, thereby encouraging optimal learning flow.

Students Explain their Reasoning

Students primarily utilize *Symphony Math* independently for individualized practice with fundamental concepts and number relationships. A teacher can also use the program with a small group of students or the entire class by projecting the program onto a screen at the front of the classroom. *Symphony Math* offers a "demonstration controller" that allows a teacher to bring up specific tasks in the program to be solved by the group of students. For example, a second grade teacher could teach a lesson on addition word problems by projecting specific tasks from *Symphony Math* on to the screen. The teacher selects a volunteer to come to the front of the class and solve the problem either by using the computer mouse or by moving the symbols and manipulatives on the screen (if an interactive white-board is available). The teacher asks the student to explain why she solved the task the way that she did. The teacher asks other students if they agree with the solution and the explanation. In this way, *Symphony Math* can be used as a tool to engage students in conversations about fundamental math concepts and challenge them to explain the reasoning. A number of researchers have called for an emphasis in math on explaining and discussing math problems.

Scope & Sequence

The scope and sequence of *Symphony Math* is organized around modules. Each module represents a fundamental network of mathematical concepts. The modules are organized hierarchically to follow the progression of the development of mathematical concepts. Each module builds on the skills and concepts of the previous module, just as later mathematical concepts are built upon previous concepts. The five *Symphony Math* modules are:

- Quantity: An understanding of the concept of numbers, that early numbers in the counting sequence represent smaller quantities and later numbers in the counting sequence represent larger quantities.
- Addition & Subtraction: An understanding of part to whole relations, that wholes are comprised of smaller parts.
- Multiplication & Division: An understanding of repeated groupings, that multiplication represents repeated addition and division represents repeated subtraction.
- Place Value: An understanding of hierarchical groupings, that larger numbers are composed of ones, tens, hundreds and thousands.
- Multi-digit Addition & Subtraction: An understanding of composing and decomposing larger numbers.

Module 1: *Quantity*

The *Quantity* module helps students develop number conceptualization. Some students can learn to count and solve simple math problems without understanding that a number represents a specific quantity. A student may know that 9 comes after 8 but not understand that 9 represents a larger quantity than 8. The student has learned the counting sequence similar to the way she learned the alphabet. However, unlike the alphabet, the sequence of counting numbers represents an increase in magnitude with each number. The sequence of numbers is determined by each number's magnitude, a concept that not all children easily understand. The *Quantity* module is designed to move children from thinking of math only as counting to understanding math as a system to represent and describe quantities. The *Quantity* module uses virtual manipulatives, symbols, and story problems to develop the following concepts:

- Number
- One-to-one correspondence
- Equality
- Greater than
- Less than
- Not equal
- Not less than
- Not greater than

Quantity also emphasizes fluency skills. Once students develop a strong understanding of a concept, they are challenged to quickly and accurately solve problems that incorporate that concept. The fluency activity in *Quantity* develops skills to solve problems with symbols, manipulatives, and auditory statements.

Another important component of the *Quantity* module is solving story problems. Story problems are presented orally for students to represent by constructing number sentences. Students learn how numbers and symbols can be used to describe real situations. For example, the student hears, “Suzy has five pencils. Jamal has three pencils.” The student is asked to represent the problem mathematically (e.g., 5 > 3).

The *Quantity* module contains twelve levels:

Level	**Concept**	**Example**
1	Equals	3 = ?
2	Greater	5 > ?
3	Less	5 < ?
4	In between	5 < ? < 9
5	Multiple solutions	5 < ? < 9 (solve three different ways)
6	Determine the nature of the relationship	5 ? 7 ? 9 (insert < or >)
7	Not equal	3 ≠ ?
8	Not greater	5 ≯?
9	Not less	5 ≮?
10	Not in between	5 ≮ ? ≯ 9
11	Negative relationships with multiple solutions	5 ≮ ? ≯9 (solve three different ways)
12	Determine the nature of the relationship (with negative relational symbols)	5 ? 7 ? 9 (insert ≮ or ≯)

The main purpose of the twelve levels is to help students understand that numbers represent quantities, or amounts, and that these amounts can be put in relationship to each other in different ways. Some amounts are equal to others, less than others, or greater than others. Levels 6 through 12 involve the use of negative relationship symbols. These challenge students to think more deeply about the nature of the relationship. It is only necessary to master the first three levels in order to activate the second module.

Module 2: *Addition & Subtraction*

The *Addition & Subtraction* module challenges students to construct a solid understanding of the fundamental concept of part-to-whole relations as well as gain mastery of basic number relationships. Many students learn to solve simple arithmetic problems, but not all children develop a conceptual understanding of what the operations mean. Elementary math problems can be solved by counting as a primary strategy. Although neither fast nor efficient, students can use counting strategies to find the correct answer. While effective in the early grades, eventually these counting strategies become too cumbersome and inefficient as the complexity of the curriculum increases. Additionally, the counting strategies by themselves do not lend themselves to a conceptual understanding of the operations.

The *Addition & Subtraction* module emphasizes the understanding of the concept of part-to-whole relations. This is the key concept that students must internalize to understand addition and subtraction at the conceptual level. The activity uses virtual manipulatives to develop the part-to-whole concept that underlies addition and subtraction. Variations of this concept includes part-to-whole, missing part, and missing parts.

The *Addition & Subtraction* module also develops procedural skills for solving addition and subtraction problems. Once students have developed a conceptual understanding of the fundamental ideas that underpin addition and subtraction, they are challenged to apply that knowledge by solving addition and subtraction problems. The module includes tasks with addends and subtrahends up to 10.

The *Addition & Subtraction* module contains nineteen levels:

Level	Concept	Example
1	Introduction to addition	2 + 1 = ?
2	Introduction to missing addend	2 + ? = 3
3	Introduction to subtraction	3 – 1 = ?
4	Introduction to missing subtrahend	3 - ? = 2
5	Intermediate addition	5 + 6 = ?
6	Intermediate missing addend	5 + ? = 11
7	Intermediate missing addends	? + ? = 11
8	Intermediate subtraction	11 – 5 = ?
9	Intermediate missing subtrahend	11 - ? = 6
10	Intermediate missing subtrahend and minuend	? - ? = 6
11	Advanced addition	9 + 8 = ?
12	Advanced missing addend	9 + ? = 17
13	Advanced subtraction	17 – 8 = ?
14	Advanced missing subtrahend	17 - ? = 9
15	Three-part addition	3 + 2 + 5 = ?
16	Three-part missing addend	3 + 2 + ? = 5
17	Missing part of sum	2 + 3 = ? + 4
18	Three-part subtraction	10 – 3 – 2 = ?
19	Three-part missing subtrahend	10 – 3 - ? = 5

The *Addition & Subtraction* module emphasizes both conceptual and fluency skills. To assist with the development of conceptual understanding, students are challenged to find multiple solutions to certain problems. A student is asked ? + ? = 10. She must then find three unique solutions to this problem. This is designed to help the student understand that different parts can be used to compose the same whole. Students are also asked to make connections between operations, in order to better understand the relationship between addition and subtraction and their connection to the part-to-whole model. For example, a student sees a series of problems with the same number relationships, but must use different operations to show how they are related. A students is asked, «?+9=10, 10-?=9, 1+?=10, 10-?=9.»

Place Value

This module is designed to help students understand the concept of hierarchical groupings that underlie the place value system. Some students can achieve correct answers on place value questions by extending their application of the counting sequence. They may be able to count to 100, or even 1,000, but not have full comprehension of how these larger numbers are structured in terms of ones, tens, hundreds and thousands.

For example, a student can answer the following question by applying the counting sequence, «which number is larger, 79 or 81?» The student determines that 81 comes after 79 and therefore answers that 81 is the larger number. This strategy produces a correct answer. But that student may not understand that 79 is composed of seven 10s and nine 1s, and that 81 is composed of eight 10s and one 1. The understanding of how these numbers are composed in the base-10 place value system is foundational to success with multi-digit addition and subtraction, where students must recompose larger numbers in addition and decompose them for subtraction.

A student who does not have a strong understanding of the concept of number and has not internalized the counting sequence to 100 may have difficulty solving the question, «which number is larger,79 or 81?» He might think 79 is larger because the 7 and the 9 combined makes 16 while the 8 and 1 combined only makes 9. This also indicates a lack of understanding that the 7 in seventy nine equals seven tens and the eight in 81 equals eight tens.

The *Place Value* module is designed to help students move past these conceptual barriers and prepare them for multi-digit addition and subtraction. The module does this by challenging students to compose and decompose numbers of increasing complexity. Students are asked to take apart and build numbers in the ones' place column. For example, 1+1+1+1=? and ?+6=10. This is essentially a review of concepts from the *Addition & Subtraction* module. The review is important because the 10s, 100s, and 1000s concepts follow this same pattern and are built upon these earlier understandings.

Understanding of the tens' place value is developed by challenging students to solve problems where they have to compose and decompose numbers such as 10, 20, 30, etc. For example, 10+10+10+10=? and ?+40=100. After succeeding in developing an understanding of the tens' place value, students progress to the hundreds: 100+100+100=? and ?+400=1000.

At this point, students have a basic understanding of what 1, 10, 100, and 1000 mean. The next step is learning how to integrate the ones', tens', and hundreds' place values. For example, 50+4=?. Some students might answer 90 or 9. This is incorrectly applying the addition skill they learned with the ones' place value. They add the 5 from 50 with the 4, not understanding that the 5 in 50 represents 5 tens. Once students become proficient at integrating two-digit numbers, they progress to three-digit numbers.

The *Place Value* module contains eighteen levels:

Level	Concept	Example
1	Intro to ones	1 + 1 + 1 + 1 + 1 = ?
2	Compose ones	4 + 5 = ?
3	Decompose ones	? + 6 = 10
4	Comparing quantity with ones	9 > ? > 7
5	Intro to tens	10 + 10 + 10 = ?
6	Compose tens	30 + 40 = ?
7	Decompose tens	40 + ? = 100
8	Comparing quantity with tens	70 < ? < 90
9	Intro to hundreds	100 + 100 + 100 = ?
10	Compose hundreds	500 + 400 = ?
11	Decompose hundreds	? + 600 = 1000
12	Comparing quantity with hundreds	800 > ? > 600
13	Composing two-digit numbers	50 + 4 + 20 + 2 = ?
14	Decomposing two-digit numbers	3 + ? + 40 + ? = 75
15	Comparing quantity with two-digit numbers	70 < ? < 83
16	Composing three-digit numbers	4 + 60 + 200 + 5 = ?
17	Decomposing three-digit numbers	? + 200 + 300 + 30 = 536
18	Comparing quantity with three-digit numbers	971 > ? > 799

This progression of understanding ones, tens, and hundreds and then integrating two- and three-digit numbers is followed through each of the five activities used in the previous modules. Students work with number bars to see concretely how five tens can be combined to make 50. They work with number bars and symbols in the second activity to understand the meaning of the symbols. In the third activity, they work with the symbols to solve number sentences but the number bars appear to provide additional support. The fourth activity helps them to learn the names of the larger numbers and how to say them. The fifth activity uses story problems to provide real-world examples of how to solve problems involving heirarchical groupings.

The are a number of pedagogical strategies in the *Place Value* module that are designed to push students' understanding to a deeper level and help them to make connections between concepts. After solving a problem such as 300+500=?, they will be asked 30+50=? and then 3+5=?. This is designed to help students associate the larger place value numbers to their mastery of addition number relationships and understanding the common patterns to these larger number relationships. Students will also encounter problems such as 971> ? > 799 in order to consolidate their emerging knowledge of place value and to explicitly connect it to the concepts they learned in the *Quantity* module where they solved problems such as 9> ? > 7.

Module 4: *Multiplication & Division*

The *Multiplication & Division* module develops an understanding of equal-sized groupings and equal sized partitioning by building upon the part-to-whole concepts learned in the *Addition & Subtraction* module. Similar to addition, some students can memorize multiplication number relationships without understanding their meaning. The *Multiplication & Division* module helps students to develop their conceptual understanding of what these operations mean and then helps them to learn the number relationships through systematic practice and evaluation.

The *Multiplication & Division* module covers number relationships with products and dividends up to thirty. The activities use the repeated addition model of multiplication and the repeated subtraction model of division. This helps students to understand the connection between the concepts in the *Addition & Subtraction* and the *Multiplication & Division* modules.

The *Multiplication & Division* module contains fourteen levels:

Level	**Concept**	**Example**
1	Introduction to multiplication	2 x 1 = ?
2	Introduction to missing multiplier	? x 2 = 2
3	Introduction to missing multiplicand	2 x ? = 2
4	Introduction to division	2 ÷ 1 = ?
5	Introduction to missing dividend	? ÷ 1 = 2
6	Introduction to missing divisor	2 ÷ ? = 2
7	Introduction to missing multiplier and multiplicand	? x ? = 2
8	Intermediate multiplication	3 x 9 = ?
9	Intermediate missing multiplier	? x 9 = 27
10	Intermediate missing multiplicand	3 x ? = 27
11	Intermediate division	27 ÷ 9 = ?

12	Intermediate missing dividend	? ÷ 9 = 3
13	Intermediate missing divisor	27 ÷ ? = 3
14	Intermediate missing dividend and divisor	? ÷ ? = 3

The *Multiplication & Division* module offers the same five activities as the other modules. In the first activity the student needs to find equal bars that are evenly related to the whole. The second activity consists of analyzing the relationship of the bars in order to construct a corresponding number sentence. The third activity presents number sentence problems typical of traditional worksheets. The number bars appear in order to present a concrete model of what the number sentence means if the student makes an error or needs help. The fourth activity offers auditory number sentences that students must construct and solve. The fifth activity presents auditory story problems such as: "James has three bags of apples. In each bag there are five apples. How many apples does James have altogether?" The student is asked to represent and solve the problem mathematically (i.e., 3x5=15).

Similar to the *Addition & Subtraction* module, this module challenges students to develop a deeper understanding of number relationships and the relationship between operations. Students are challenged to find up to three unique solutions for problems such as ?x?=12 and ?÷?=12.

Module 5: *Multi-Digit Addition & Subtraction*
The *Multi-Digit Addition & Subtraction* module consolidates the concepts developed in the previous modules and extends them into the complex process of adding and subtracting large numbers that require regrouping and decomposing of specific place values. This process is dramatically more complex both in terms of the necessary conceptual understanding and the number of steps it requires to produce the correct solution. In this way, it is unlike the previous concepts that precede it and can serve as a barrier to some children in their mathematical learning.

The module uses the traditional algorithm of solving from right to left starting with the ones column. This is not the only way to solve these types of problems, but it is the most widely accepted and is effective with every type of problem. Students can also be encouraged to learn alternative solutions in their classroom. Since not all classrooms embrace the teaching of alternative algorithms, *Symphony Math* does not introduce them. The common critique of the traditional algorithm is that it is not intuitive and some students employ it in a rote manner without a grasp of its meaning. *Symphony Math* addresses this weakness of the traditional algorithm by introducing it through the five *Symphony Math* activities; manipulatives, manipulatives and symbols, symbols, auditory statements, and story problems. While the process is clearly complex, the use of the manipulatives in Activity One helps students intuitively relate to the logic behind the traditional algorithm. Students can «see» why they must bring the ten over from the ones to the tens column in addition and why they need to break down a hundred to bring over some tens to the tens column in subtraction.

The *Multi-Digit Addition & Subtraction* module contains twenty-four levels:

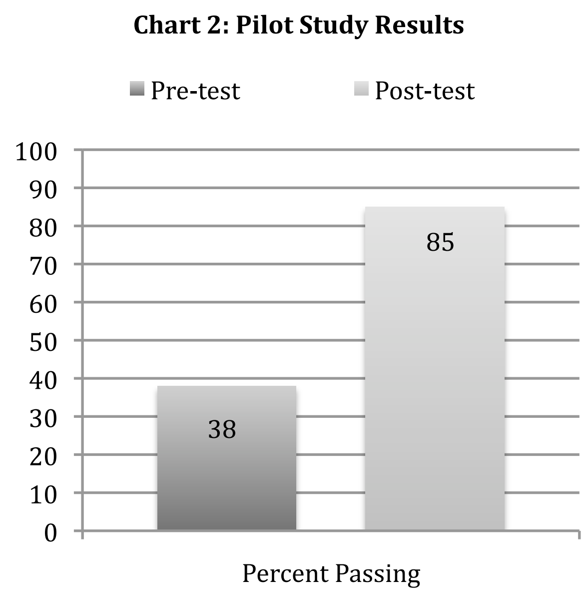
Level	**Concept**	**Example (in vertical format)**
1	Adding one-digit numbers	3 + 4 = ?
2	Adding two-digit numbers	23 + 31 = ?
3	Adding three-digit numbers	325 + 271 = ?
4	Missing addend one-digit numbers	3 + ? = 9
5	Missing addend two-digit numbers	23 + ? = 65
6	Missing addend three-digit numbers	237 + ? = 989
7	Subtracting one-digit numbers	9 – 2 = ?
8	Subtracting two-digit numbers	87 – 34 = ?
9	Subtracting three-digit numbers	638 – 214 = ?
10	Missing subtrahend one-digit numbers	8 - ? = 2
11	Missing subtrahend two-digit numbers	45 - ? = 12
12	Missing subtrahend three-digit numbers	628 - ? = 210
13	Adding one-digit numbers with recomposing	9 + 8 = ?
14	Missing addend one-digit numbers with recomposing	8 + ? = 15
15	Adding two-digit numbers with recomposing	45 + 87 = ?
16	Missing addend two-digit numbers with recomposing	32 + ? = 123
17	Adding three-digit numbers with recomposing	342 + 879 = ?
18	Missing addend three-digit numbers with recomposing	453 + ? = 124
19	Subtraction one-digit numbers with decomposing	13 – 9 = 4
20	Missing subtrahend one-digit numbers with decomposing	14 - ? = 9
21	Subtraction two-digit numbers with decomposing	132 – 87 = ?
22	Missing subtrahend two-digit numbers with decomposing	142 - ? = 86
23	Subtraction three-digit numbers with decomposing	1321 – 487 = ?
24	Missing subtrahend three-digit numbers with decomposing	1423 - ? = 578

The *Multi-Digit Addition & Subtraction* module begins with addition and subtraction problems that do not require recomposing (carrying) and decomposing (borrowing). These early levels of the module introduce students to the vertical format of addition and subtraction. The previous modules only used the horizontal model of addition and subtraction. In order to develop a solid understanding of larger number relationships, students are challenged to solve missing addend and missing subtrahend problems such as 237 + ? = 989 and 628 - ? = 210.

Addition with recomposing is introduced with simple one-digit addends that were previously in the *Addition & Subtraction* module. Familiarity with these number relationships helps students to develop an understanding of recomposing numbers in the vertical addition format. Students progress to adding two-digit numbers with recomposing. Initially, only one of the two columns involves recomposing. At the end of the level, both columns involve recomposing. Three-digit addition follows a similar pattern where students are introduced to the level with only one column requiring recomposing. With correct responses, students progress to problems requiring recomposing in two columns and eventually three columns. Subtracting large numbers with decomposing follows a similar progression.

Pilot Study

A pilot study was conducted at an elementary school in Florida with 13 students. All of the students were limited in their English language proficiency. The students were pre-tested on a number knowledge test designed by the school staff. Eight of the thirteen students failed the pre-test (scored less than 66% correct). The students used *Symphony Math* for thirteen 15-minute sessions in the computer lab as part of their math instruction time. After using *Symphony Math* for an average of 3 hours per student, the post-test was administered. On the post-test only 2 of the thirteen students who took the test failed.



Conclusion

Symphony Math is an educational software program that is designed to overcome some of the more challenging barriers to improving math education. *Symphony Math* is specifically targeted to address the most fundamental mathematical concepts. This helps teachers who are burdened with a broad and superficial curriculum by focusing on the most important concepts. *Symphony Math* helps students improve their understanding of concepts as well as number relationship fluency; thereby sidestepping the "math wars" debate that dictates that only one or the other should be emphasized. *Symphony Math* is designed to be an easily-implemented technology solution. The program is fully automated and adapts to the needs of each student. This enables teachers to implement a complex research-based curriculum with little or no specific training. Preliminary results from an early pilot study are encouraging. More rigorous and larger-scale studies will be the focus of future research.

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