stage 17:
add & subtract unit fractions

Big Idea: Repeated Equal Groupings with Parts-to-Whole

The big idea of Repeated Equal Groupings with Parts-to-Whole is the most complex of the big ideas addressed so far. Similar to Repeated Equal Groupings this big idea involves repeating an equal sized group, or partitioning an amount into equal groups. It also involves a trio of parts to the whole. Not only do students have to keep track of the whole and its equal groups as with multiplication and division, but they also have to be mindful of the number of parts relative to the whole.

For example, with multiplication and division we work with the total, the size of the parts, and the number of parts. There are 30 students in the class (the whole equals 30), the teacher divides the class into 5 groups (the number of parts equals 5), and there are six students in each group (the size of the parts equals 6). This leads us to 30 divided by 5 equals 6.

Let's take the same Equal Groupings sample and add the Parts-to-Whole component. What if the teacher said that 1/5 of the class had siblings in lower grades: how can we determine how many students have siblings in lower grades? The fraction of one-fifth is a parts-to-whole representation. It means one part out of five parts. To determine how many students is equal to 1/5 of 30 we can first use Equal Groupings. Five equal groups of 30 means that there are 6 students in each group. Therefore, 1/5 of 30 equals 6.

It is for this reason that an in-depth understanding of Repeated Equal Groupings with Parts-to-Whole is fundamental to understanding fractions. Students need to understand that the big idea of Parts- to -Whole is fundamental to understanding fractions. Students need to understand the big idea of Parts-to-whole developed with addition and subtraction as well as the big ideas of Repeated Equal Groupings developed with multiplication and division. These two big ideas coordinated together give us Repeated Equal Groupings with Parts-to-Whole, the foundational idea for fractions. This perspective also helps us understand why fractions can be so difficult for students. Not only do they need to have mastery of the proceeding big ideas, but they need to coordinate them together.

Why are Repeated Equal Groupings and Parts-to-Whole Important?

Because of the complexity of fractions, and other related concepts such as ratios, decimals and percents, students need to understand the coordination of Equal Groupings with Part-to-Whole. An understanding of this big idea will help students better navigate the many counter-intuitive and confusing aspects of fractions.

Fractions are difficult for students to learn not only because of its conceptual complexity but also because many of its features appear contradictory to what students have learned previously regarding whole numbers. For example, up to now in their experience with whole numbers, students have learned that two distinct numbers indicate different amounts. The concept of equivalence in fractions indicates that 2/2 equal one whole, and 4/4 equal 2/2, and so on. They have leaned that 2 does not equal 1, 4 does not equal 2. But with fractions 2/2 does equal 1, and these equivalent fractions are the same point on a

number line! In fact, this is the first time in their experience that numbers do not signal a count. Rather, fractional numbers are a part relative to a whole.

A further confusing idea may be that experiences multiplying whole numbers up to now yielded bigger amounts. When dividing, smaller amounts resulted. But when multiplying proper fractions, smaller amounts result, and when dividing with proper fractions, the result is bigger. The before-learned properties of how numbers behave does not apply to fractional numbers. Equally confusing is the fact that fractions with the same numerals can equal different amounts. A student might expect that 1/2 always equals 1/2. But the student must consider 1/2 of what? One half of $100 does not equal 1/2 of $50.

Students may also find the names of fractions to be confusing. Previously students have understood late numbers in the counting sequence to represent larger quantities. Twelve is greater than 2 for example. However with fractions, 1/12 is not greater than 1/2.

When some students are confronted with the complexity of fractions and they do not have sufficient mastery of the necessary big ideas they may resort to memorized strategies that lead to correct answers. They learn rules like “find the common denominator, then add.” Or, “flip it and multiply.” They might remind themselves that “the bigger the denominator the smaller the fraction.” While these memorized strategies may lead to correct answers, on their own they do not lead to deep understanding of fractions.

Symphony Math continues with fraction concepts by insuring students construct knowledge based on ideas of magnitude and numeracy, rather than on procedural steps.

Stage 17 Learning Progression

In Stage 17, Symphony Math expands on the four concepts that comprise the Equal Groupings with Parts-to-Whole big idea:

* the whole is divided
* there are a specific number of parts
* the parts are of equal size
* the parts equal the whole

In Stage 17, Symphony Math introduces fractions with denominators up to 10. Using the same rich visual environment as Stage 17, students continue to experience a systematic foundation on which to build their understanding of Equal Groupings with Parts to Whole.

In addition to making equivalencies and comparisons with different numerators and different denominators, students add and subtract with fractions. In Stages 17.3-17.6 students are challenged to find missing parts, missing wholes, and missing relationships. By solving problems from the perspective of missing parts, missing wholes, and missing relationships, students come to a better understanding of the Equal Groupings with Parts-to-Whole big idea.

The Symphony Math visual learning environment explicitly connects adding and subtracting fractions with students’ prior experiences composing and decomposing quantities.

Concept	Standard	Example	Description
17.1: Equivalent Fractions	4.NF.1	1/2 = ?/8.	Students see that the pieces that represent a fraction in a fraction bar can be further subdivided; the amount doesn't change, but the way we name the fraction does. Fractions with different names can also claim the identical point on a number line; it is the result of subdividing jumps into smaller (or larger) proportional jumps.
17.2: Comparing Fractions	4.NF.2	3/5 ? 4/7	Fraction magnitudes are key to an understanding of fractions. As students increase their deep understanding of fraction magnitudes, they will be more successful with fraction computation. In Stage 17.2, the method students use to determine the magnitude of fractions is first focused on visual fraction models: the area model and the number line model which have consistently been part of their fraction environment in Symphony Math.
17.3: Addition with Fractions: Missing Result	4.NF.3a	1/4 + 1/4 = ?	Students experience the action of combining unit fractions to make a non-unit fraction. Just as 1 + 1 + 1 = 3, 1/4 + 1/4 + 1/4 = 3/4. Addition of fractional parts actively unfold in the fraction bar + number line milieu, making addition of unit fractions explicit. Students see what happens when several copies of the unit fractions are added together.
17.4: Addition with Fractions: Missing Change	4.NF.3a	1/4 + ? = 2/4	In Stages 17.3-17.6 students are challenged to find missing parts, missing wholes, and missing relationships. By solving problems from the perspective of missing parts, missing wholes, and missing relationships, students come to a better understanding of the Equal Groupings with Parts-to-Whole big idea
17.5: Subtraction with Fractions: Missing Result	4.NF.3a	2/4 - 1/4 = ?	As stated above, continue to work with Parts-to-Whole with fractions. In Stage 17.5, students take away from the whole to produce a smaller part.
17.6: Subtraction with Fractions: Missing Change	4.NF.3a	2/4 - ? = 2/4	Students visualize the decomposition of non-unit fraction into unit fractions, helping them understand the amount of change they need to make up the difference in a missing change problem.

Using the Extra Practice Worksheets

The Symphony Math Worksheets provide extended practice using the Multiples Ways of Knowing from the Symphony Math program. Students should work through all worksheets in the order given:

Worksheet	Purpose	Instructions
Manipulatives	Use a visual model to represent the concept.	Create bars, dot cards, or number lines for each item.
Bridge	Connect symbols to their visual representations.	Create objects, numbers, and symbols to complete each item.
Symbols	Understand the concept at the abstract level.	Create numbers and symbols to complete each item.
Apply	Extend understanding to real-life problem solving.	1) Read the story presented at the top of the page. 2) Create a number model of the full solution. 3) Write the number sentence that matches the model.

Group Learning

The Symphony Math Extra Practice materials are designed to promote a conversation about the Big Ideas in math. One-on-one or small group instruction with the materials is recommended for students who need more time to make connections between the mathematical concepts in the Stage and the application of those concepts in their math curriculum.

Fraction Bars