Big Idea: Hierarchical Groupings with Parts-to-Whole

Hierarchical Groupings with Parts to Whole is a complex idea-one that involves the coordination of two earlier big ideas. It also illustrates the hierarchal nature of mathematics and how a poor foundation is likely to interfere with the learning of later concepts. Hierarchical Groupings is the idea that amounts can be grouped into a system of sets and subsets. These sets can be combined to form a whole, an amount that is equal to the sum of its parts.

Building on Stage 12, students in Stage 21 now possess the mathematical underpinnings to be able to apply their place value understanding of numbers as they engage the standard algorithm process for multi-digit addition and subtraction. The vertical representation, introduced in Stage 7, shows place value columns in which the potentially complex procedure for addition and subtraction makes sense.

Why is Parts to Whole so Important?

The student who has not developed an in-depth understanding of these big ideas is likely to rely only on the step-by-step application of procedures without the supporting understanding of what these procedures mean and why they work. When he produces a nonsensical answer, the mistake is not apparent because he has stopped looking at the numbers as representing meaningful quantities. He is simply manipulating symbols as best he can according to that procedure.

With the groundwork established especially in Stages 7 - 12, students understand the structure of tens, hundreds, and thousands and can thus apply their knowledge in a way that makes sense as they use the standard algorithm for the first time, supported by Symphony visual environments.

Stage 21 Learning Progression

Concept	Standard	Example	Description
21.1: 2-digit and 1-digit Addition	4.NBT.4	87 + 9 = ?	Students work with the vertical place value presentation and see how to solve the problem using the standard algorithm process. Dot-cards provide familiar visual models for regrouping in the standard way. Combining 2-digit numbers with 1-digit numbers is a recognized task for students and thus a safe entry point for the introduction of standard algorithm procedure.
21.2: 2-digit and 1-digit Subtraction	4.NBT.4	87 – 9 = ?	Students work with the vertical place value presentation and see how to solve a subtraction problem using the standard algorithm process. Dot-cards provide familiar visual modes for decomposing in the standard way. Subtracting a 1-digit number from a 2-digit number is a recognized task for students and thus a safe entry point for the introduction of the standard algorithm.
21.3: 2-digit and 2-digit Addition	4.NBT.4	58 + 67 = ?	Students work with the vertical place value presentation and see how to solve a double-digit addition problem using the standard algorithm process. Regrouping ones into tens and tens into hundreds are actions students encounter in multiple formats in the Symphony Math program. Those experiences provide students with mathematical flexibility and understanding which forms the foundation that allows them to appreciate the efficiency of the standard algorithm.
21.4: 2-digit and 2-digit Subtraction	4.NBT.4	82 – 59 = ?	Students work with the vertical place value presentation and see how to solve a double-digit subtraction problem using the standard algorithm process. Decomposing tens into ones is an action that students encounter in multiple formats in the Symphony Math program. Those experiences provide students with mathematical flexibility and under-standing which forms the foundation that allows them to appreciate the efficiency of the standard algorithm.
21.5: 3-digit and 3-digit Addition	4.NBT.4	368 + 848 = ?	In Stage 12.6, students regrouped two 3-digit numbers by composing and decomposing tens, making a 100, breaking apart 10, or creating an easier problem. In Stage 21.5, students can see the efficiency of the standard algorithm as they follow and make sense of the procedures entailed.
21.6: 3-digit and 3-digit Subtraction	4.NBT.4	848 – 368 = ?	In Stage 12.7, students demonstrate their knowledge of hundreds, tens and ones and how such parts decompose to make a new whole. In Stage 21.6, students apply this knowledge when they subtract two 3-digit numbers using the steps involved in the standard algorithm for subtraction.
21.7: 3-Part Addition with Mixed Digits	4.NBT.4	368 + 78 = ?	Students use the standard algorithm for addition to compute the sum of three addends of 1-, 2-, and 3-digits, necessitating that they use their place value knowledge to add and regroup accordingly. Consistent with Stage 21, numbers are presented in the vertical column format and accompanied by a visual environment.
21.8: 4-digit Subtraction	4.NBT.4	1,470 – 258 = ?	Multi-digit subtraction using the standard algorithm poses a challenge for students. Students prior experiences decomposing large quantities come into practice now as they see the vertical presentation of the problem: With the myriad tasks Symphony Math provides for students in earlier Stages, supported by visuals, the standard subtraction algorithm may be appreciated for its simplicity.

Using the Extra Practice Worksheets

The Symphony Math Worksheets provide extended practice using the Multiples Ways of Knowing from the Symphony Math program. Students should work through all worksheets in the order given:

Worksheet	Purpose	Instructions
Manipulatives	Use a visual model to represent the concept.	Create bars, dot cards, or number lines for each item.
Bridge	Connect symbols to their visual representations.	Create objects, numbers, and symbols to complete each item.
Symbols	Understand the concept at the abstract level.	Create numbers and symbols to complete each item.
Apply	Extend understanding to real-life problem solving.	1) Read the story presented at the top of the page. 2) Create a number model of the full solution. 3) Write the number sentence that matches the model.

Group Learning

The Symphony Math Extra Practice materials are designed to promote a conversation about the Big Ideas in math. One-on-one or small group instruction with the materials is recommended for students who need more time to make connections between the mathematical concepts in the Stage and the application of those concepts in their math curriculum.

Dot Cards