stage 23:
multiplying fractions and wholes

Big Idea: Repeated Equal Groupings with Parts-to-Whole

Students need to understand that the big idea of Parts- to -Whole is fundamental to understanding fractions. In addition, students need to understand the big idea of Parts-to-whole developed with addition and subtraction as well as the big ideas of Repeated Equal Groupings developed with multiplication and division. These two big ideas coordinated together give us Repeated Equal Groupings with Parts-to-Whole, the foundational idea for fractions. This perspective also helps us understand why fractions can be so difficult for students. Not only do they need to have mastery of the proceeding big ideas, but they need to coordinate them together. Students are called on to extend their understanding of whole number multiplication and how fractions behave when they multiply these two quantities together.

Why are Repeated Equal Groupings so Important?

Because of the complexity of fractions, and other related concepts such as ratios, decimals and percents, students need to understand the coordination of Equal Groupings with Part-to-Whole. An understanding of this big idea will help students better navigate the many counter-intuitive and confusing aspects of fractions.

Fractions are difficult for students to learn not only because of the conceptual complexity but also because many of the features appear contradictory to what students have learned previously regarding whole numbers. For example, with whole numbers, students learn that when multiplied, their product is larger than either of the number or size of groups with which they begin. Initially, fraction multiplication appears inconsistent with what students know.

With repeated equal groupings of fractional amounts, 3 x 2/5, the product will be smaller than the number of groups [3] with which they begin. In the situation 3 x 2/5, we can ask how many group sizes of 2/5s are in 3 groups? It takes 6 (1/5s) to make 3 groups, or 1 1/2 [2/5 groups]. The answer, 1 1/2, in isolation, is a smaller amount than 3, because it is not obvious that the quantity of 1 1/2 tells us how many group sizes of 2/5s we are talking about.

In Stage 23, students are grounded by virtue of the visual fraction models that make Symphony Math. In addition to the number line, students see a model of fraction bars atop a number line, introduced in Stage 17. Using visual models, students see the pictorial representation of those groups of 2/5 size, and then what happens when 3 of the groups are repeated. Thus Symphony Math helps students grapple with the significance and meaning of 'the whole' which can be elusive when dealing with fraction operations.

Stage 23 Learning Progression

Concept	Standard	Example	Description
23.1: Whole Numbers x Unit Fractions	5.NF.4	1/4 x 32 = ?	In Stage 17, students learn that 4/5 = 1/5 + 1/5 + 1/5 + 1/5. A natural extension of understanding non-unit fractions is that 4/5 also equals 4 x 1/5. The fraction bar/number line models show students that fraction parts can be counted, just as numbers are counted. If 4 cookies can be thought of as 4 groups of 1 cookie, so to can 4/5 be shown as 4 groups of 1/5 or 4 x 1/5. In this way, SM links students' prior experiences multiplying whole numbers with multiplying fractions.
23.2: Whole Numbers x Non-Unit Fractions	5.NF.4	5 x 3/5 = ?	To multiply non-unit fractions by a whole number, students extend their understanding of multiples of 1/b, Substage 23.1, and see that group sizes of a/b are also repeatedly added by the number of groups being considered: 4 x (2/7) = 2/7 + 2/7 +2/7 + 2/7= 8/7 = 8 x (1/7) = 1 1/7. Visual fraction models allow students to make sense of the product of a whole number times a non-unit fraction. Modeling number line jumps, or fraction bar delineations, of [2/7] being repeated 4 times, make a robust pictorial for students. Students will be able to make their own similar models when they multiply non-unit fractions by whole numbers.
23.3: Fractions x Whole Numbers: Missing Part	5.NF.4	2/3 x ? = 18	Students may see "How many groups of 1/4 make 2 1/4? 8/4? Stage 23.3 employs different fraction formats, (mixed numbers, fractions with numerators larger than their denominators and fractions with larger denominators than numerators), and shows students that they repeatedly add fractional groups of any size a certain number of times until they arrive at the desired product. By watching number line jumps and fraction bar increases of specified group-sizes, students can see when they land on their target number. The problem structures of Stage 23.3 is more challenging than 23.2, yet the connection between multiplication and division, made in Stage 13, come to bear.

Using the Extra Practice Worksheets

The Symphony Math Worksheets provide extended practice using the Multiples Ways of Knowing from the Symphony Math program. Students should work through all worksheets in the order given:
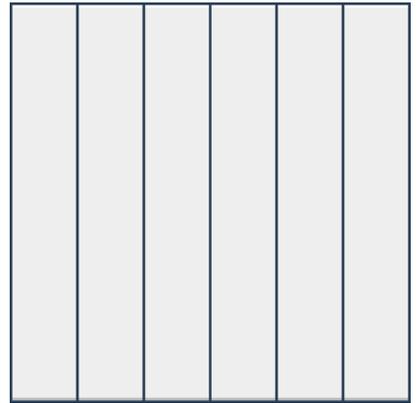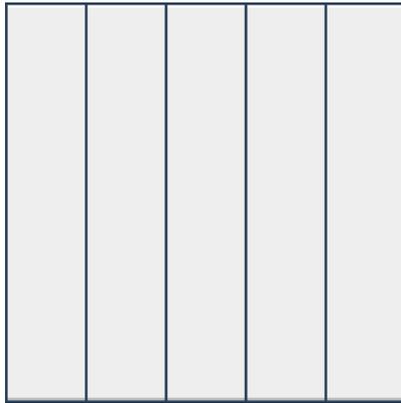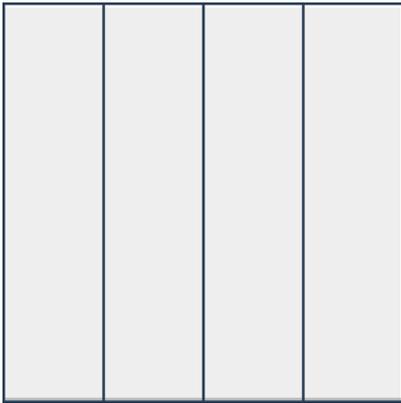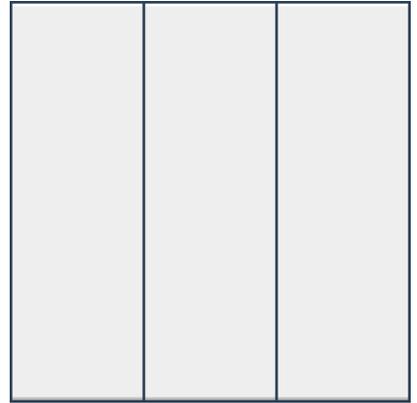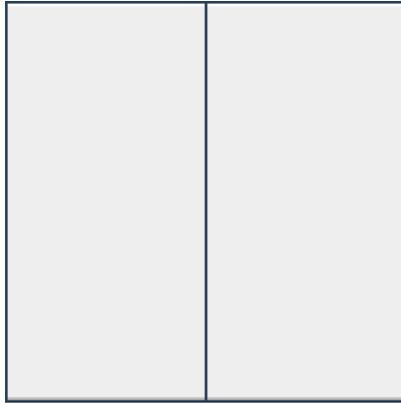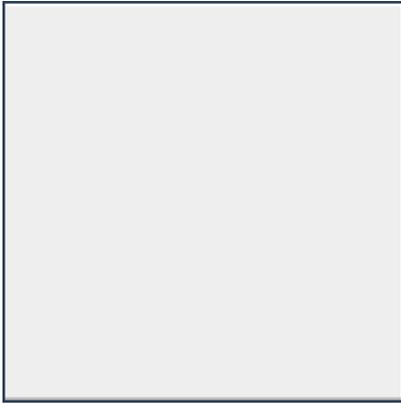
Worksheet	Purpose	Instructions
Manipulatives	Use a visual model to represent the concept.	Create bars, dot cards, or number lines for each item.
Bridge	Connect symbols to their visual representations.	Create objects, numbers, and symbols to complete each item.
Symbols	Understand the concept at the abstract level.	Create numbers and symbols to complete each item.
Apply	Extend understanding to real-life problem solving.	1) Read the story presented at the top of the page. 2) Create a number model of the full solution. 3) Write the number sentence that matches the model.

Group Learning

The Symphony Math Extra Practice materials are designed to promote a conversation about the Big Ideas in math. One-on-one or small group instruction with the materials is recommended for students who need more time to make connections between the mathematical concepts in the Stage and the application of those concepts in their math curriculum.

Fraction Bars