stage 8:
addition & subtraction with 10s

Big Idea: Hierarchical Groupings with Parts-to-Whole

As its name implies, Hierarchical Groupings with Parts-to-Whole is a complex idea -- one that involves the coordination of two earlier big ideas. It also illustrates the hierarchal nature of mathematics and how a poor foundation is likely to interfere with the learning of later concepts. For this big idea students must coordinate their knowledge of Hierarchal Groupings and Parts-to-Whole. Hierarchical Groupings is the idea that amounts can be grouped into a system of sets and subsets. These sets can be combined to form a whole, an amount that is equal to the sum of its parts.

Why is this So Important?

Stages 1-7 in Symphony Math emphasize combinations to 20, and then the underlying structure of numbers to 100. Before students move on to numbers beyond 100, they must develop an understanding of quantity, the composition of numbers and their inter-relatedness, and the organization of tens and ones. Such underpinnings are vital to a student's ability to flexibly and fluently apply organization and meaning to future work and operations with larger numbers and fraction and decimal numbers.

In Stage 8, Symphony Math extends the idea of ten as a unit to multiple groups of ten. Ten is a benchmark number, and combinations with multiples of ten are scaffolded in such way to elucidate how 'tens and some more' form the numbers 10-100. Stage 8 extends the behavior of ten as a benchmark number in order to compose and decompose numbers 10-100. Making tens is a key strategy children are encouraged to use; and decade numbers behave much as one ten does-make as many bundles of ten as possible, and then add on the leftovers.

Stage 8 Learning Progression

Concept	Standard	Example	Description
8.1: Place Value Addition: Missing Result	1.OA.8	20 + 3 = ?	Symphony Math models provide the visual manipulatives in which groups of ten are distinguishable from groups of ones. When a student shows she is ready not to have to count all the ones, combining tens and some ones becomes automatic; looking at a visual representation of 27, she is able to count 2 tens, and know the 2 references 2 bundles of ten or twenty. The leftover ones appear physically different than the representation of tens. Thus the total becomes increasingly automatic.
8.2: Place Value Addition: Missing Change	1.NBT.4	20 + ? = 23	Given the whole, the student uses what she's learned regarding the relationship between addition and subtraction, making groups of ten, and hierarchical groupings in order to find the missing part. She can complete the part that makes the equation between the three whole numbers true, thus showing that she sees the relationship of the three numbers to each other.
8.3: Place Value Subtraction: Missing Result	1.OA.8	23 - 3 = ?	As her experiences with combinations of parts and wholes continues, as well as her grasp on the structure of ten and groups of ten, the student can work on 8.3 with good understanding. She is asked to think of subtracting ones from a given group of tens combined with that same number of ones (47 - 7). Symphony Math's visual tools model how groups of tens and some ones leave those same groups of tens, once the ones are taken away. (47 - 7 = 40)

8.4: Place Value Subtraction: Missing Change	2.OA.1	23 - ? = 20	Given the whole, the student uses what she's learned regarding the relationship between addition and subtraction, making groups of ten, and hierarchical groupings in order to find the missing part. She can complete the part that makes the equation between the three whole numbers true, thus showing that she sees the relationship of the three numbers to each other.
8.5: Parts-to-Whole with 1s and 10s	2.NBT.5	56 = ? + ? + 6	Stage 8.5 challenges the student to decompose a 2-digit number into varying parts of tens and ones, in a multitude of ways. Based on place value, properties of operations, and the relationship between addition and subtraction, she has the chance to put her knowledge of number composition to the test. Given a number 73, she may need to find 2 missing addends + 3 that combine to make the whole; or 2 additional missing addends + 30 to compose that same whole.
8.6: Parts-to-Whole with 1s and 10s (+10)	1.NBT.5	69 + 10 = ?	Given any 2-digit number, students are asked to show they can add 1 ten more. Stage 7.6 highlights how ten is not an ordinary number; they are not adding 1 more, but an additional 1 group of ten. The significant properties of ten make a difference to the structure of the original number. The visual tools in Symphony Math make the results of this action apparent, thus omitting the need to count to find the answer. Adding ten becomes a mental strategy.
8.7: Parts-to-Whole with 1s and 10s (-10)	1.NBT.5	56 - 10 = ?	Given any 2-digit number, students are asked to show they can take away 1 ten. As in Stage 8.6, ten is highlighted as a special number, one that behaves differently than subtracting 1. With the initial aid of visual manipulatives, students can subtract 10 and see the results without counting. 'Minus 10' is a developmentally sophisticated strategy students use to subtract. Through the examples in Symphony Math, subtracting ten becomes an action students can do mentally.
8.8: Adding With Multiples of 10-No Regrouping	1.NBT.4	61 = ? + 21	Students work on their automaticity with composing with multiples of tens. They use number relationships learned in earlier stages so that such combinations become fluent. Stage 8.8 furthers previous work with the concept of adding tens and tens and ones and ones. Eventually adding multiples 10 becomes a key mental strategy.
8.9: Comparing 2-Digit Numbers	1.NBT.2	67 > ?	Students compare two two-digit numbers based on meanings of the tens and ones digits, and show the results using symbols >, =, <. They use their understanding of the hierarchical order of groups, and the order of digits to compare two sets of numbers based on each's amounts of tens and ones.

Using the Extra Practice Worksheets

The Symphony Math Worksheets provide extended practice using the Multiples Ways of Knowing from the Symphony Math program. Students should work through all worksheets in the order given:
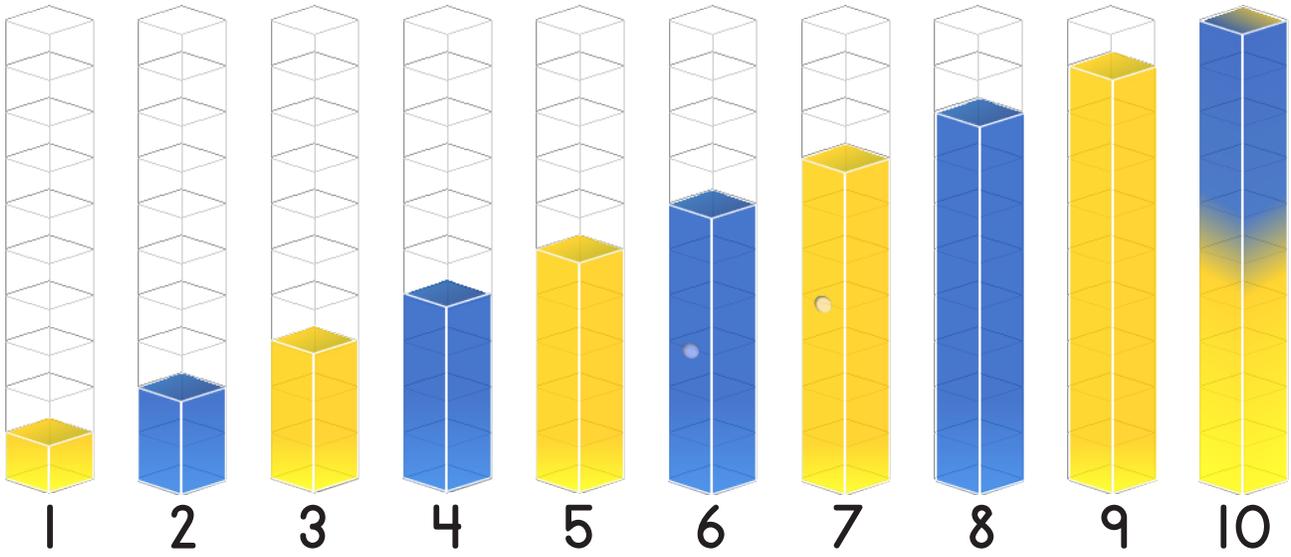
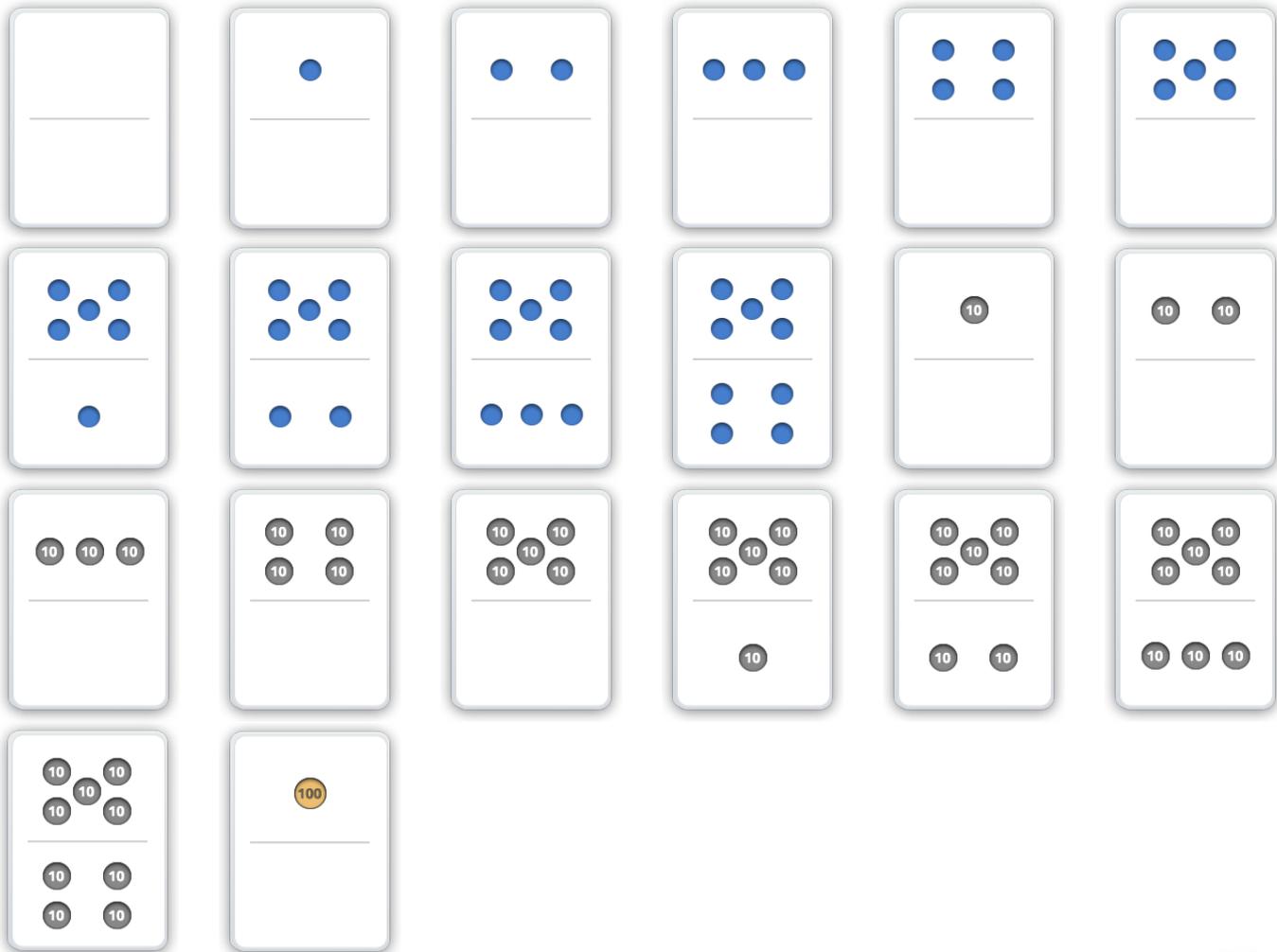| Worksheet	Purpose	Instructions
Manipulatives	Use a visual model to represent the concept.	Create bars, dot cards, or number lines for each item.
Bridge	Connect symbols to their visual representations.	Create objects, numbers, and symbols to complete each item.
Symbols	Understand the concept at the abstract level.	Create numbers and symbols to complete each item.
Apply	Extend understanding to real-life problem solving.	1) Read the story presented at the top of the page. 2) Create a number model of the full solution. 3) Write the number sentence that matches the model.

Group Learning

The Symphony Math Extra Practice materials are designed to promote a conversation about the Big Ideas in math. One-on-one or small group instruction with the materials is recommended for students who need more time to make connections between the mathematical concepts in the Stage and the application of those concepts in their math curriculum.
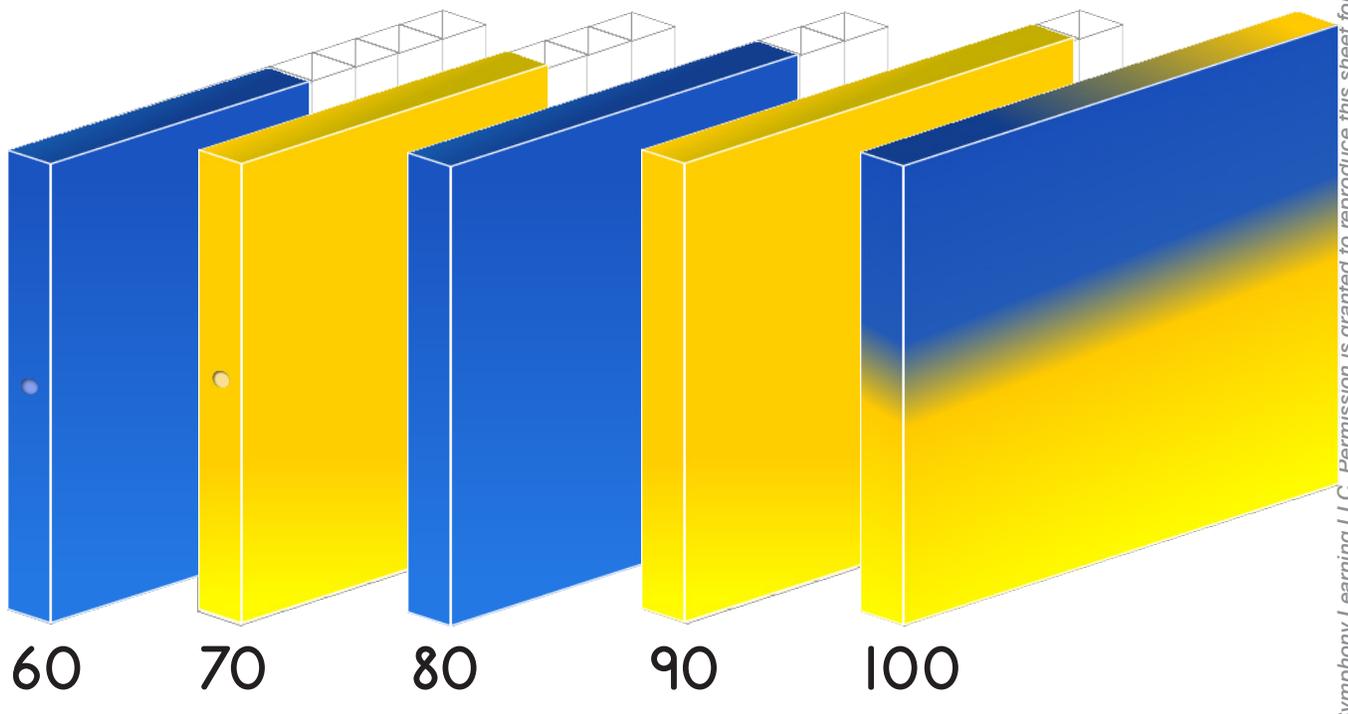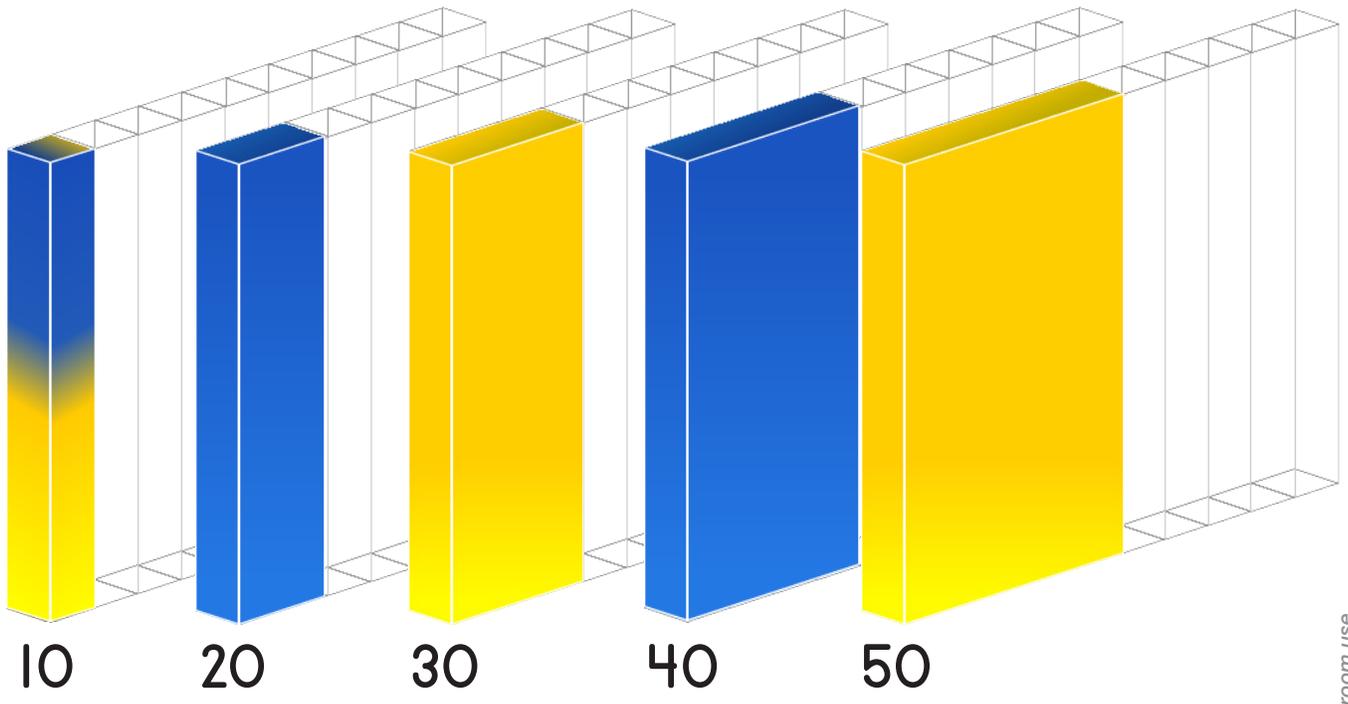
Symphony Bars

1   2   3   4   5   6   7   8   9   10

Dot Cards

10　20　30　40　50

60　70　80　90　100