stage 15:
multiply & divide to 100

Big Idea: Repeated Equal Groupings

The Repeated Equal Groupings big idea builds upon the Parts-to-Whole idea. With Repeated Equal Groupings, the whole is not only broken into parts but broken into a specific number of parts and each part is of equal size. For example, there are 30 students in the class (the whole). The teacher divides the class into groups of 6 (the equal groups). These equal groups are repeated 5 times to equal the whole.

Repeated equal groupings is the big idea that underlies multiplication and division. Multiplication consists of taking a part (the multiplicand) and repeating it a certain number of times (the multiplier) to equal the whole (the product). Division consists of a whole (the dividend) that is partitioned into a certain number of equal groups (the divisor) that is equal to the size of the parts (the quotient).

Why are Repeated Equal Groupings Important?

It is possible for students to memorize multiplication facts without understanding what they mean. Students can also use skip counting to correctly solve multiplication problems without appreciating the significance of how they arrived at the correct answer. In addition to being able to immediately recall multiplication and division facts, students also need to be able to understand what these operations mean.

Students who understand equal groupings have a better chance of memorizing multiplication and division number relationships because they have a conceptual basis to support their learning. For example, they are more likely to see the connections between, 7 x 4, 4 x 7, 28 ÷ 7, and 28 ÷ 4 . Stages 11 and 13 develop an understanding of grouping and partitioning by building on the Parts-to-Whole concepts established in Stages 3, 6, 8, and 10. They reinforce the concept of repeated groupings-that multiplication represents repeated addition and division represents repeated subtraction. Previous Symphony Math Stages help students develop their conceptual understanding of what these operations mean, and then help students learn the number relationships through systematic practice and evaluation.

In addition to further practice with multiplication and division to 100, Symphony Math goes beyond the rote ‘memorize-tables’ approach. Number relationships are key in order that students observe and experience how numbers behave in relation with each other. When students work through a problem like 5 x 12 = 60, they will see that 12 x 5 = 60, and that 5 x 10 + 5 x 2 = 60 as well as that 60 ÷ 5 = 12 and so on.

Stage 15 Learning Progression

Concept	Standard	Example	Description
15.1: Multiplication to 100	3.OA.3	7 x 9 = ?	Unknown Product is the first problem structure in Stage 15. Students interpret 3 x 7 as the total number of objects in 3 groups of 7 objects each, for example. They learn to recognize multiplication as a way to determine the total number of objects when there are a specific number of groups with the same number of objects in each group.
15.2: Multiplication: Unknown Factor / Group Size to 100	3.OA.3	? x 9 = 63	The problem structures of 15.2 and 15.3 are progressively more challenging than that of finding an unknown product. Because students have experienced many instances of the equal sign up to this point, they may think, on seeing ? x 4 = 12, that some number of groups of 4 is the same as 15.
15.3: Multiplication: Unknown Number of Groups to 100	3.OA.4	7 x ? = 63	Where 15.2 focuses on a missing size of the equal groupings, 15.3 emphasizes an unknown number of groups, or the factor.
15.4: Division: Missing Result to 100	3.OA.4	63 ÷ 9 = ?	In their introduction to division, it is important that students see a connection between multiplication and division. Partitioning models focus on how many objects are in each group when the groups are equal. As students work on problems in 15.4, they begin to see how the 'fact family' relationships learned in work with multiplication are applicable to division situations in an inverse way.
15.5: Missing Dividend to 100	3.OA.6	63 ÷ ? = 7	15.5 is an opportunity for students to use their knowledge of multiplication and its connection to division. Missing dividend problems are another way of asking them to multiply the two given factors to find a product.
15.6: Missing Divisor to 100	3.OA.6	? ÷ 9 = 7	Students determine the unknown by relating the 3 numbers in the same 'fact family' to each other to make the visual representations and then equations true.
15.7: Multiplication and the Commutative Property to 100	3.OA.5	12 x 5 = 5 x ?	Students were first introduced to the commutative property of addition in Stage 3. The same rule applies to multiplication. As students work on tasks that show 2 x 3 = 6 and 3 x 2 = 6, they begin to see the constancy in the quantities and subsequently, the relationship these 3 numbers have to each other.
15.8: Multiplication and the Distributive Property to 100	3.OA.5	6x9 + 3x9 = ?	Students break up numbers into smaller numbers. The distributive property can be a way for students to both find an easier way to compute, as well as show their understanding of how smaller parts behave when multiplying.

Using the Extra Practice Worksheets

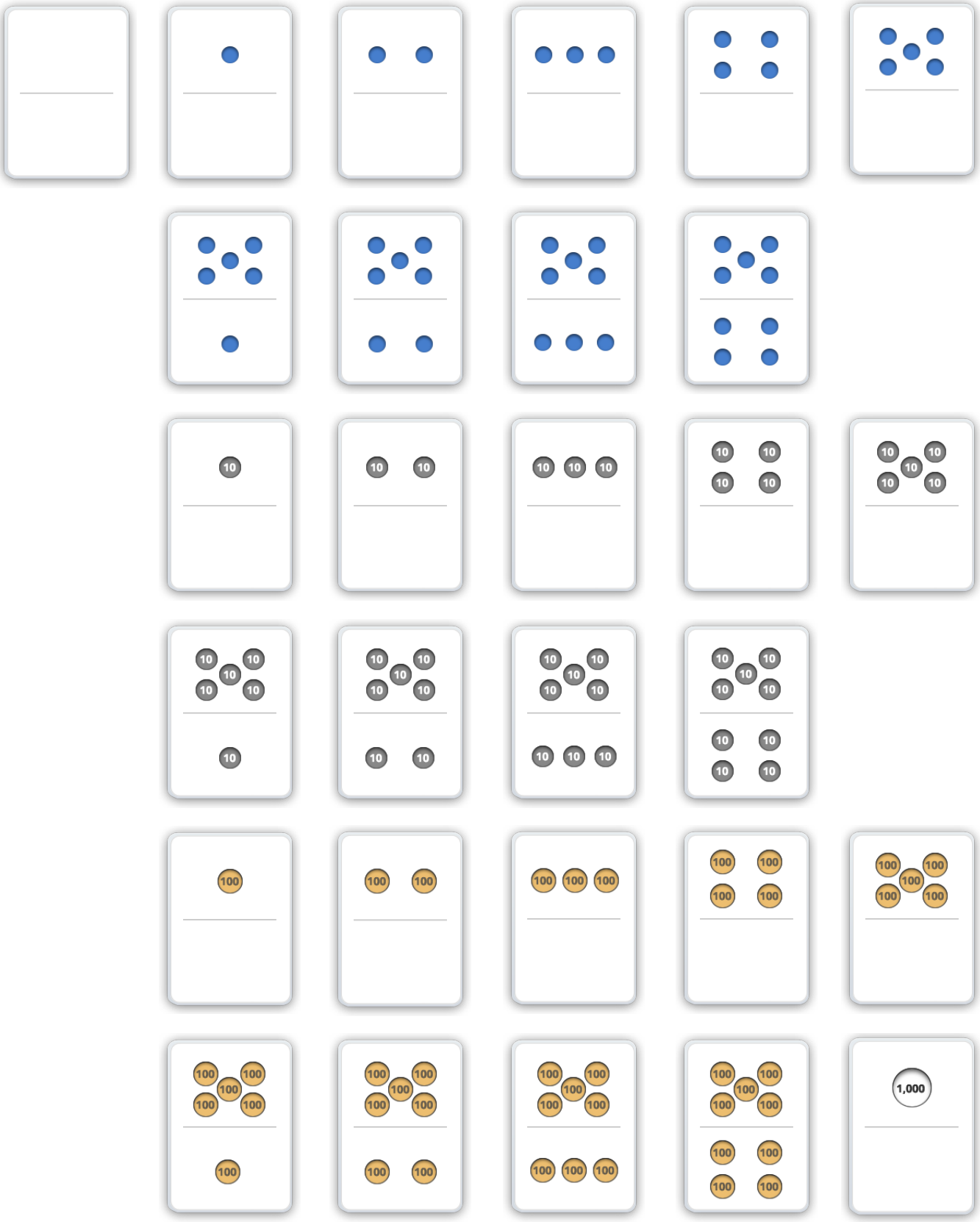
The Symphony Math Worksheets provide extended practice using the Multiples Ways of Knowing from the Symphony Math program. Students should work through all worksheets in the order given:

Worksheet	Purpose	Instructions
Manipulatives	Use a visual model to represent the concept.	Create bars, dot cards, or number lines for each item.
Bridge	Connect symbols to their visual representations.	Create objects, numbers, and symbols to complete each item.
Symbols	Understand the concept at the abstract level.	Create numbers and symbols to complete each item.
Apply	Extend understanding to real-life problem solving.	1) Read the story presented at the top of the page. 2) Create a number model of the full solution. 3) Write the number sentence that matches the model.

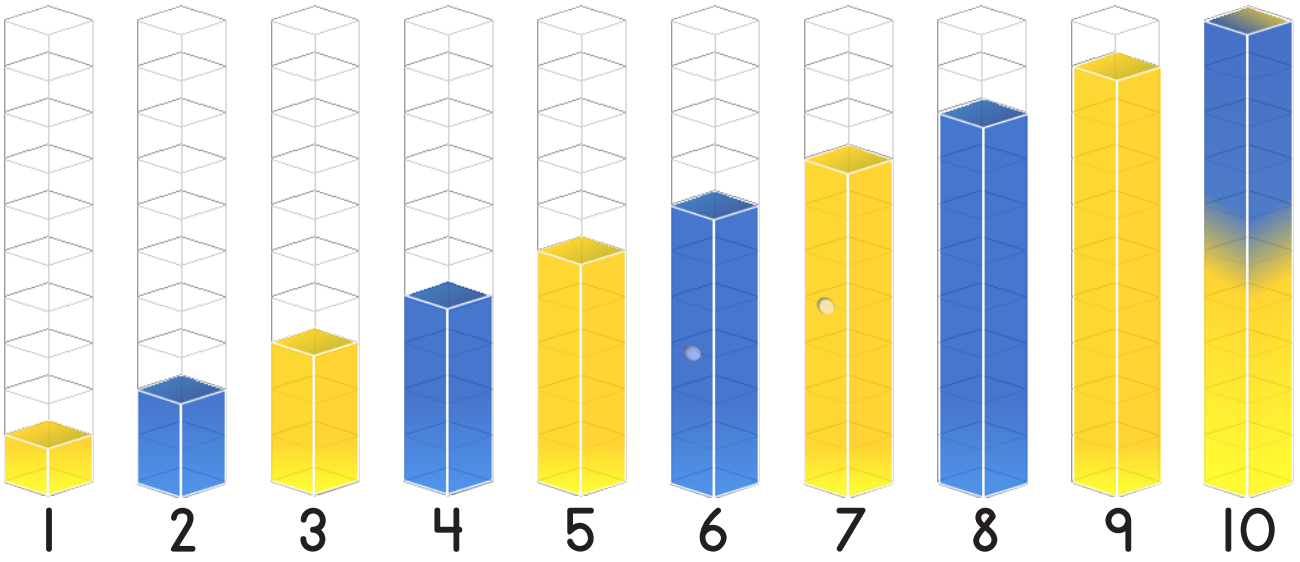
Group Learning

The Symphony Math Extra Practice materials are designed to promote a conversation about the Big Ideas in math. One-on-one or small group instruction with the materials is recommended for students who need more time to make connections between the mathematical concepts in the Stage and the application of those concepts in their math curriculum.

Dot Cards



Symphony Bars: Ones & 10



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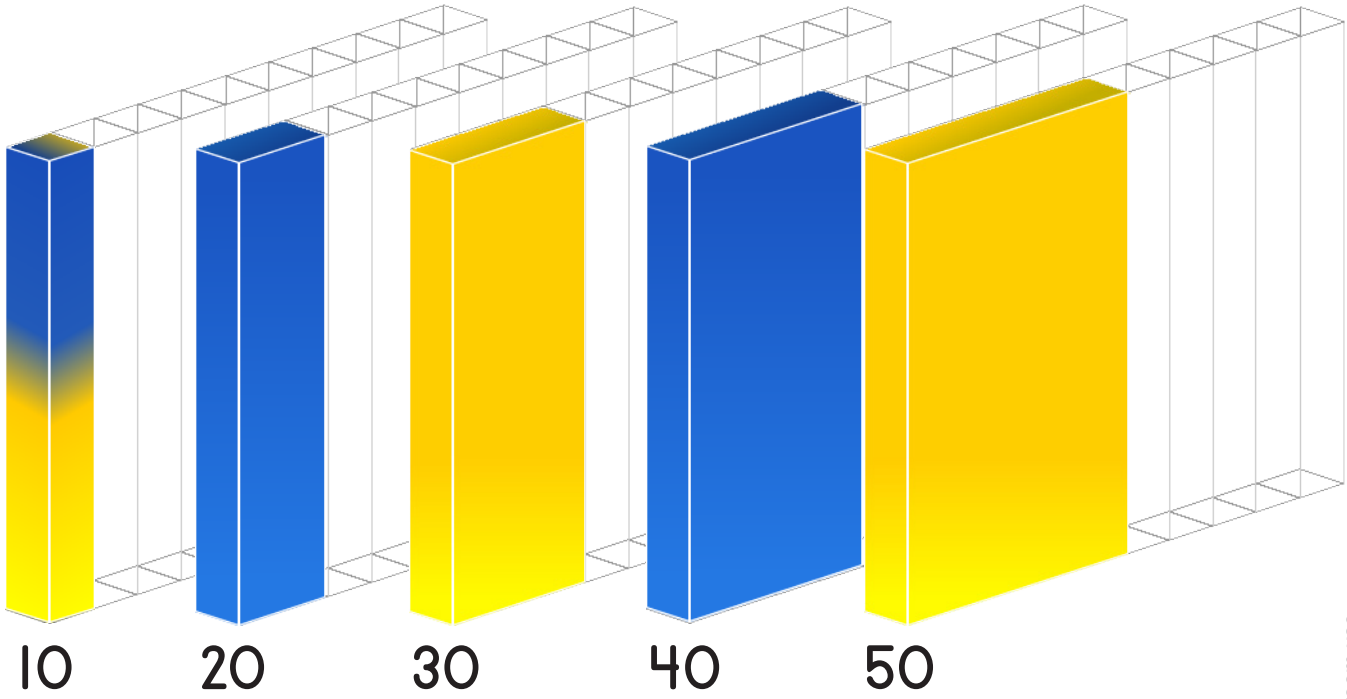

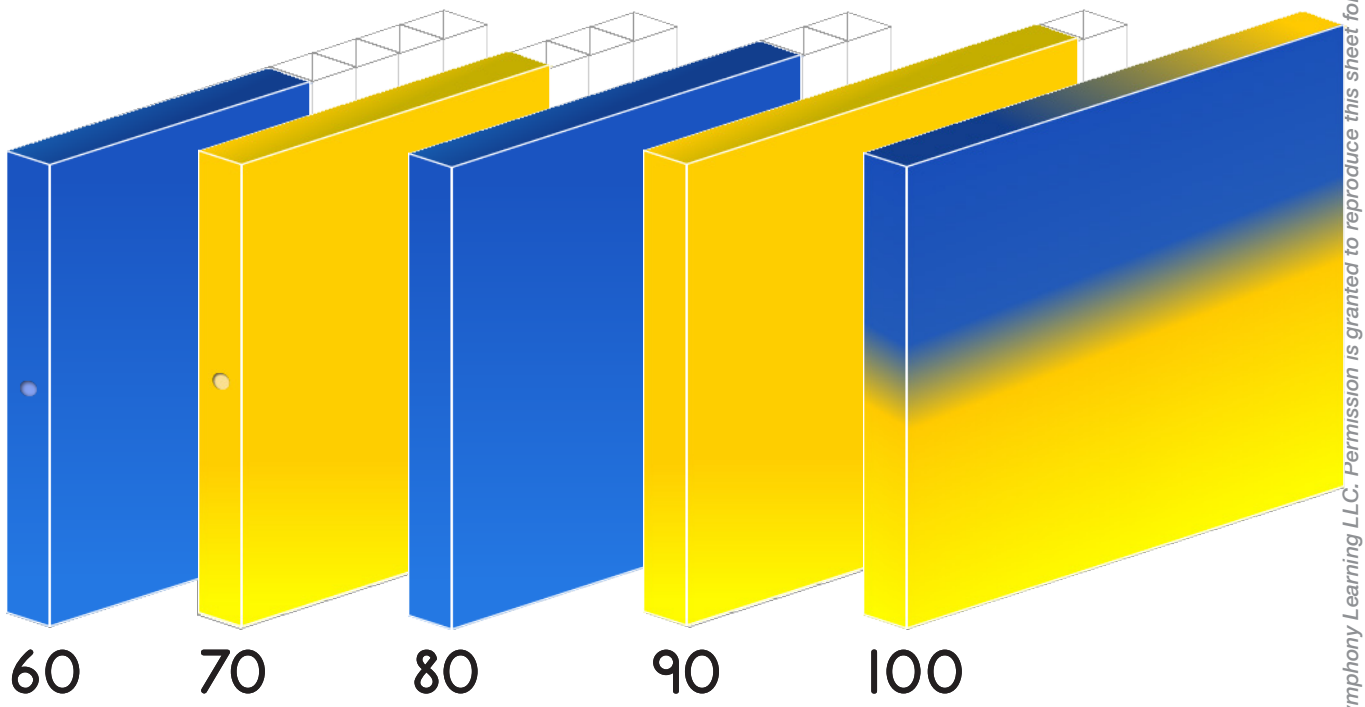

Symphony Bars: Hundreds