stage 18:
non-unit fractions

Big Idea: Repeated Equal Groupings with Parts-to-Whole

The big idea of Repeated Equal Groupings with Parts-to-Whole is the most complex of the big ideas addressed so far. Similar to Repeated Equal Groupings this big idea involves repeating an equal sized group, or partitioning an amount into equal groups. It also involves a trio of parts to the whole. Not only do students have to keep track of the whole and its equal groups as with multiplication and division, but they also have to be mindful of the number of parts relative to the whole.

For example, with multiplication and division we work with the total, the size of the parts, and the number of parts. There are 30 students in the class (the whole equals 30), the teacher divides the class into 5 groups (the number of parts equals 5), and there are six students in each group (the size of the parts equals 6). This leads us to 30 divided by 5 equals 6.

Let's take the same Equal Groupings sample and add the Parts-to-Whole component. What if the teacher said that 1/5 of the class had siblings in lower grades: how can we determine how many students have siblings in lower grades? The fraction of one-fifth is a parts-to-whole representation. It means one part out of five parts. To determine how many students is equal to 1/5 of 30 we can first use Equal Groupings. Five equal groups of 30 means that there are 6 students in each group. Therefore, 1/5 of 30 equals 6.

It is for this reason that an in-depth understanding of Repeated Equal Groupings with Parts-to-Whole is fundamental to understanding fractions. Students need to understand that the big idea of Parts- to -Whole is fundamental to understanding fractions. Students need to understand the big idea of Parts-to-whole developed with addition and subtraction as well as the big ideas of Repeated Equal Groupings developed with multiplication and division. These two big ideas coordinated together give us Repeated Equal Groupings with Parts-to-Whole, the foundational idea for fractions. This perspective also helps us understand why fractions can be so difficult for students. Not only do they need to have mastery of the proceeding big ideas, but they need to coordinate them together.

Why are Repeated Equal Groupings and Parts-to-Whole Important?

Because of the complexity of fractions, and other related concepts such as ratios, decimals and percents, students need to understand the coordination of Equal Groupings with Part-to-Whole. An understanding of this big idea will help students better navigate the many counter-intuitive and confusing aspects of fractions.

Fractions are difficult for students to learn not only because of its conceptual complexity but also because many of its features appear contradictory to what students have learned previously regarding whole numbers. For example, up to now in their experience with whole numbers, students have learned that two distinct numbers indicate different amounts. The concept of equivalence in fractions indicates that 2/2 equal one whole, and 4/4 equal 2/2, and so on. They have leaned that 2 does not equal 1, 4 does not equal 2. But with fractions 2/2 does equal 1, and these equivalent fractions are the same point on a

number line! In fact, this is the first time in their experience that numbers do not signal a count. Rather, fractional numbers are a part relative to a whole.

A further confusing idea may be that experiences multiplying whole numbers up to now yielded bigger amounts. When dividing, smaller amounts resulted. But when multiplying proper fractions, smaller amounts result, and when dividing with proper fractions, the result is bigger. The before-learned properties of how numbers behave does not apply to fractional numbers. Equally confusing is the fact that fractions with the same numerals can equal different amounts. A student might expect that 1/2 always equals 1/2. But the student must consider 1/2 of what? One half of $100 does not equal 1/2 of $50.

Students may also find the names of fractions to be confusing. Previously students have understood late numbers in the counting sequence to represent larger quantities. Twelve is greater than 2 for example. However with fractions, 1/12 is not greater than 1/2.

When some students are confronted with the complexity of fractions and they do not have sufficient mastery of the necessary big ideas they may resort to memorized strategies that lead to correct answers. They learn rules like “find the common denominator, then add.” Or, “flip it and multiply.” They might remind themselves that “the bigger the denominator the smaller the fraction.” While these memorized strategies may lead to correct answers, on their own they do not lead to deep understanding of fractions.

Symphony Math continues with fraction concepts by insuring students construct knowledge based on ideas of magnitude and numeracy, rather than on procedural steps.

Stage 18 Learning Progression

In Stage 17 students experienced how copies of unit fractions add together to create non-unit fractions, (where numerators are greater than 1). Stage 18 continues the work begun in Stage 18, with the computation of non-unit fractions with results less than or equal to 1 whole.

In this way, students remain in a challenging but familiar environment before they work with fractions more than 1 whole (improper fractions and mixed numbers). Thus they will be on familiar ground when Stage 20 introduces these concepts for the first time.

Concept	Standard	Example	Description
18.1: Addition with Non-Unit Fractions: Missing Result	4.NF.3b	2/5 + 3/5 =?	Students use their experiences with combining unit fractions from Stage 17.3 to combine non-unit fractions. In Stage 18, students also call on the structure of whole number addition, and their growing fluency with number combinations. Just as 3 + 5 = 8, 3/9 + 5/9 = 8/9.
18.2: Addition with Non-Unit Fractions: Missing Change	4.NF.3b	2/6 + ? = 5/6	Students use their experiences with combining unit fractions from Stage 17.3 and 17.4 to combine non-unit fractions. When the result of that combination is known, students supply the missing parts that combine to make that sum. In Stage 18.2, students also call on the structure of whole number addition, and their growing fluency with number combinations. Just as 3 + ? = 8, 3/9 + ?/9 = 8/9, and the missing part is 5, or 5/9.
18.3: Subtraction with Non-Unit Fractions: Missing Result	4.NF.3b	3/3 - 2/3 = ?	Students use their experiences combining unit fractions from Stage 17.5 to decompose a fraction amount into its parts and determine what is left. In Stage 18.3, students also call on the structure of whole number subtraction, and their growing fluency with number combinations. Just as 8 - 3 = 5, 8/9 - 3/9 = 5/9.
18.4: Subtraction with Non-Unit Fractions: Missing Change	4.NF.3b	7/8 - ? = 3/8	Students use their experiences combining unit fractions from Stage 17.5 and 17.6 to decompose a fraction amount into its parts and determine what is missing. In Stage 18.4, students also call on the structure of whole number subtraction, and their growing fluency with number combinations. Just as 8 - ?= 5, 8/9 - ?/9 = 5/9.with the missing change being 3 or 3/9.

Using the Extra Practice Worksheets

The Symphony Math Worksheets provide extended practice using the Multiples Ways of Knowing from the Symphony Math program. Students should work through all worksheets in the order given:

Worksheet	Purpose	Instructions
Manipulatives	Use a visual model to represent the concept.	Create bars, dot cards, or number lines for each item.
Bridge	Connect symbols to their visual representations.	Create objects, numbers, and symbols to complete each item.
Symbols	Understand the concept at the abstract level.	Create numbers and symbols to complete each item.
Apply	Extend understanding to real-life problem solving.	1) Read the story presented at the top of the page. 2) Create a number model of the full solution. 3) Write the number sentence that matches the model.

Group Learning

The Symphony Math Extra Practice materials are designed to promote a conversation about the Big Ideas in math. One-on-one or small group instruction with the materials is recommended for students who need more time to make connections between the mathematical concepts in the Stage and the application of those concepts in their math curriculum.

Fraction Bars