



# stage 20:

## fractions greater than 1 whole

### Big Idea: Repeated Equal Groupings with Parts-to-Whole

Stage 20 is the first time students experience fractions greater than 1 whole. They see such quantities in what is commonly called 'improper fraction' format as well as in mixed number format. While students have undoubtedly seen and used terms like '5 and  $\frac{1}{2}$ ' or '3 and  $\frac{3}{4}$ ,' they have not necessarily understood that eleven halves, or three wholes + another three quarters is the same quantity as  $5\frac{1}{2}$  and  $3\frac{3}{4}$  respectively.

Students need to understand that the big idea of Parts-to-Whole is fundamental to understanding fractions. Students need to understand the big idea of Parts-to-whole developed with addition and subtraction as well as the big ideas of Repeated Equal Groupings developed with multiplication and division. These two big ideas coordinated together give us Repeated Equal Groupings with Parts-to-Whole, the foundational idea for fractions. This perspective also helps us understand why fractions can be so difficult for students. Not only do they need to have mastery of the preceding big ideas, but they need to coordinate them together.

Just as students develop foundational numeracy throughout their Symphony Math experiences, Stage 20 guides them through familiar and increased contact with fractions. And because of their underpinnings with fractions, fraction composition and decomposition is more of an extension of their past exposure rather than a stretch.

### Why are Repeated Equal Groupings so Important?

Because of the complexity of fractions, and other related concepts such as ratios, decimals and percents, students need to understand the coordination of Equal Groupings with Part-to-Whole. An understanding of this big idea will help students better navigate the many counter-intuitive and confusing aspects of fractions.

Fractions are difficult for students to learn not only because of its conceptual complexity but also because many of its features appear contradictory to what students have learned previously regarding whole numbers. For example, up to now in their experience with whole numbers, students have learned that two distinct numbers indicate different amounts. The concept of equivalence in fractions indicates that  $\frac{2}{2}$  equal one whole, and  $\frac{4}{4}$  equal  $\frac{2}{2}$ , and so on. They have learned that 2 does not equal 1, 4 does not equal 2. But with fractions  $\frac{2}{2}$  does equal 1, and these equivalent fractions are the same point on a number line! In fact, this is the first time in their experience that numbers do not signal a count. Rather, fractional numbers are a part relative to a whole.

When some students are confronted with the complexity of fractions and they do not have sufficient mastery of the necessary big ideas they may resort to memorized strategies that lead to correct answers. They learn rules like "find the common denominator, then add." Or, "flip it and multiply." They might remind themselves that "the bigger the denominator the smaller the fraction." While these memorized strategies may lead to correct answers, on their own they do not lead to deep understanding of fractions.



Symphony Math continues with fraction concepts by insuring students construct knowledge based on ideas of magnitude and numeracy, rather than on procedural steps.

The new material in Stage 20 presents students with fractions where the whole that is greater than one. Up to now, students' experience in Symphony Math's fraction work has been with parts of 1 whole. Whereas a whole greater than one is nothing new for students, the way the whole looks and the way it is composed, when using fractions, will be explored. Students scaffold from fraction addition in Stages 17 and 18 to combinations that create mixed numbers and numerators larger than denominators (often called "improper fractions"). Their prior work decomposing fractions into sums of unit fractions, Stage 17, and working with whole numbers as fractions from Stage 14, have built the foundations on which students' work with fractions greater than 1 whole is founded.

## Stage 20 Learning Progression

Concept	Standard	Example	Description
20.1: Composing Fractions Greater Than 1 Whole	4.NF.3c	Make '3/2'	<p>In Stage 14, students used unit fractions to compose both non-unit fractions and whole number amounts. In Stage 18, students continued the progression of fraction addition and further experienced the behavior of fractions when they are combined.</p> <p>In Stage 20, students maintain this natural progression to combine fractions with like denominators with results that are greater than 1 whole. The visual model will aid their comprehension that it is possible to have a fraction with a numerator greater than its denominator and help students make sense of an understandable quantity in this newly written form.</p>
20.2: Composing Mixed Numbers	4.NF.3c	Make '2 1/3'	<p>Mixed numbers are introduced for the first time in Stage 20.2. Because Symphony Math provides students with a firm underpinning in both whole number combinations and fraction combinations, students are able to apply and extend these experiences.</p> <p>In Substages 14.2 and 14.3, students created both unit and non-unit fractions. In Stage 17 and Stage 18 they added and subtracted fractions. In Substage 20.2 students continue to have visual models to reference such prior experiences and extend them to create mixed numbers. Visual modeling continues to provide a powerful tool to help students turn improper fractions into mixed numbers.</p> <p>It is important that students understand that <math>2 \frac{1}{4}</math> is the same amount as <math>2 + \frac{1}{4}</math>, as well as 9 fourths. The Symphony Math environments contribute to this fundamental understanding.</p>
20.3: Decomposing Fractions Greater Than 1 Whole	4.NF.3c	$5/3 = 2/3 + ?$	<p>With a problem such as <math>5/3 - 2/3 = ?</math>, students can call on the structure of whole number subtraction, and their growing fluency with number combinations to understand the similarity to subtracting <math>5 - 3</math>, much as they did with fraction subtraction in Stage 18.3. Students use their experiences combining unit fractions from Stage 17.5 to decompose a fraction amount into its parts and determine what is left. The fact that the numerator is greater than the denominator does not change the behavior of the fraction parts, as students will see. The emphasis continues to be on fraction decomposition into its parts.</p>
20.4: Decomposing Mixed Numbers	4.NF.3b	$2 \frac{1}{3} = 3/3 + ?$	<p>Students show they understand that a mixed number behaves like a whole number in that it can be decomposed and recombined in different ways and still maintain the same whole. The quantity <math>2 \frac{1}{3}</math> can be achieved with <math>3/3 + 3/3 + 1/3</math> or <math>1 + 2/3 + 2/3</math> for examples. Students can reference similar work from Stage 10 where they were asked to recognize the relationships of parts that exist in any number they are working with and flexibly apply this understanding when adding and subtracting. In Stage 20.4, students are challenged to analyze the result of number decomposition in order to successfully compute the answer in a variety of ways that may seem uncommon.</p> <p>Because students have extensive experiences with part-part-whole, the idea that any quantity can be expressed as the sum of any of its parts is normal. Fraction numbers behave like whole numbers in this way. In Stage 20.4, students demonstrate their knowledge of how fraction parts can be regrouped into an equivalent whole by showing that they recognize many different parts, the equivalency of the parts, and how hierarchical structures come into play.</p>

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## Using the Extra Practice Worksheets

The Symphony Math Worksheets provide extended practice using the Multiples Ways of Knowing from the Symphony Math program. Students should work through all worksheets in the order given:

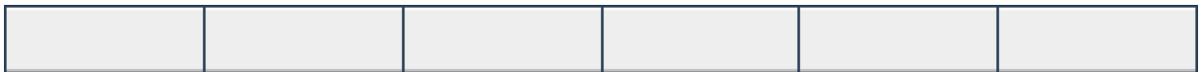
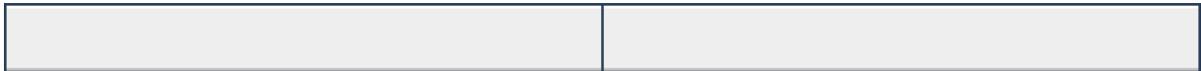
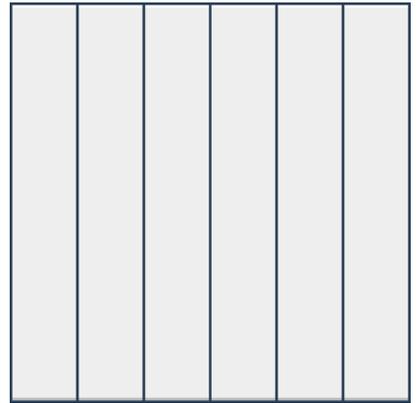
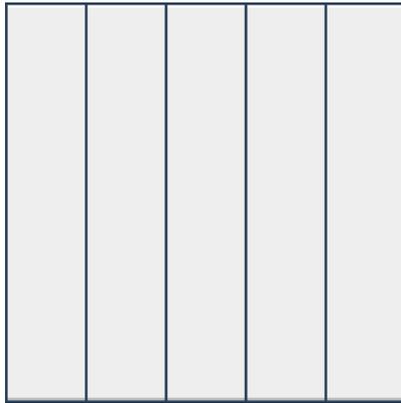
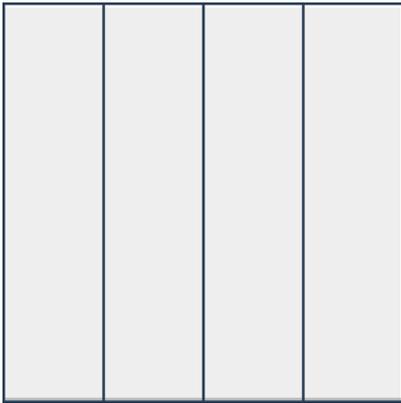
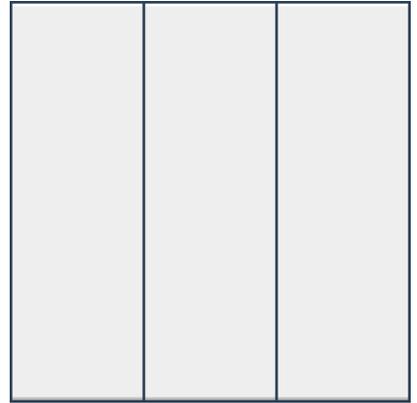
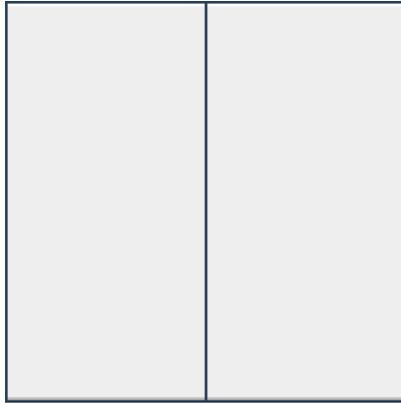
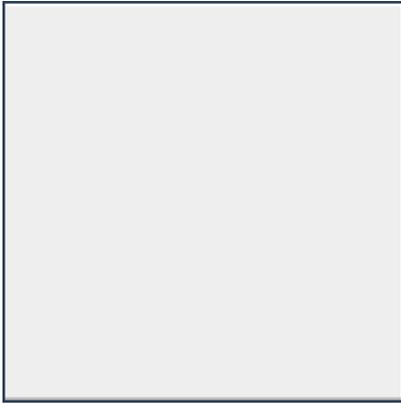
Worksheet	Purpose	Instructions
Manipulatives	Use a visual model to represent the concept.	Create bars, dot cards, or number lines for each item.
Bridge	Connect symbols to their visual representations.	Create objects, numbers, and symbols to complete each item.
Symbols	Understand the concept at the abstract level.	Create numbers and symbols to complete each item.
Apply	Extend understanding to real-life problem solving.	<ol style="list-style-type: none"><li>1) Read the story presented at the top of the page.</li><li>2) Create a number model of the full solution.</li><li>3) Write the number sentence that matches the model.</li></ol>

## Group Learning

The Symphony Math Extra Practice materials are designed to promote a conversation about the Big Ideas in math. One-on-one or small group instruction with the materials is recommended for students who need more time to make connections between the mathematical concepts in the Stage and the application of those concepts in their math curriculum.



# Fraction Bars



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